# Strike while the Iron is Hot – Optimal Monetary Policy under State-Dependent Pricing

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**Abstract**

We characterize optimal monetary policy under state-dependent price setting. The framework gives rise to nonlinear inflation dynamics: The flexibility of the price level increases after large shocks due to an endogenous rise in the frequency of price changes, as observed during the 2022-2023 inflation surge. In response to large cost-push shocks, optimal policy leverages the lower sacrifice ratio to curb inflation and stabilize the frequency of price adjustments. When faced with total factor productivity shocks, an efficient disturbance, the optimal policy commits to strict price stability. The optimal long-run inflation rate is just above zero.

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# 1. Introduction

What is the optimal design of monetary policy during an inflation surge? The traditional answer offered by the New Keynesian literature relies on a model of price setting, that of Calvo (1983), which disregards endogenous variation in the frequency of price changes: firms update prices at random times irrespective of macroeconomic conditions.[1] In contrast, an emerging literature on state-dependent price setting recognizes that firms endogenously decide when to adjust prices. The recent inflationary surge has provided empirical support for these models, as the frequency of price changes increased more than twofold during the period of high inflation.[2] However, the normative aspects of these state-dependent models have received little attention. To bridge this important gap, our paper characterizes optimal monetary policy under state-dependent price setting.

Our analysis arrives at a novel insight: optimal policy leans against inflation disproportionately strongly in response to large cost-push shocks, which push the repricing frequency endogenously high – a "strike while the iron is hot" policy. The primary reason is that the cost of the anti-inflationary policy in terms of output is smaller when the frequency of price changes increases in response to the shocks – as the price level becomes more flexible, the sacrifice ratio falls. Furthermore, as we show analytically, optimal policy requires full inflation stabilization after total factor productivity shocks – a "divine coincidence" result after efficiency shocks, as in the canonical Calvo model.

Our baseline state-dependent price setting model closely follows the seminal paper of Golosov and Lucas (2007).[3] In the model, a representative household consumes a continuum of differentiated goods, and supplies labor in a centralized, frictionless market. Each consumption good is produced by a single firm with labor as the only input. Production technology is subject to aggregate productivity and cost-push shocks, and idiosyncratic quality shocks.[4] Firms must incur a small, fixed, "menu cost" to adjust their prices. Thus, firms' pricing decisions are characterized by an $(S, s)$ rule: When prices are within an endogenous band around the optimal reset price, firms keep them constant; otherwise, they pay the menu cost and update their price. The central bank sets the nominal interest rate.

We study the optimal design of monetary policy in the model. To this end, we propose a new algorithm to solve the Ramsey problem non-linearly, so that it is suitable for assessing the impact of large aggregate shocks. In particular, we approximate the value and distribution functions over the endogenously determined relevant range and solve the set of equilibrium conditions under perfect foresight over the sequence space. We calibrate the model parameters to match the frequency of the price changes

[1] Woodford (2003); Galí (2008)

[2] For evidence, see Montag and Villar (2023); Cavallo et al. (2023); Blanco et al. (2024a)

[3] We show robustness using the Calvo-plus model of Nakamura and Steinsson (2010).

[4] We depart from the Golosov and Lucas (2007) model in this regard, which, instead of idiosyncratic quality shocks, assumes productivity shocks. This facilitates the computation, while its implications are innocuous (see also Midrigan 2011; Alvarez et al. 2021).

in the U.S. before the inflation surge[5], as well as a 20% frequency level accompanying a 10% inflation rate as experienced during the inflation surge of 2022-2023.[6] We contrast the implications of our state-dependent model to those of a time-dependent Calvo model.[7]

The model economy is subject to three welfare relevant distortions. The first two are caused by actual markups deviating from the efficient markup: the first distortion is caused by the *average* markup; and the second by the *dispersion* of markups. The third distortion is the resource costs associated with price adjustment. The average markup distortion is conventional and is present both in our baseline model and in the canonical Calvo model.[8] It incentivizes the central bank to minimize the variation, caused by aggregate shocks, in the average markup, which it can affect due to price rigidities. The behavior of the second and third distortions is distinct in the two frameworks. In our framework, aggregate shocks can reduce markup dispersion on impact, as new adjusters are *selected* from those with the most misaligned markups; while aggregate shocks increase markup dispersion in the Calvo framework. Furthermore, resource costs of price changes become a relevant factor in our framework, while they are always zero in the Calvo framework by construction. The policy maker's task is to minimize the effect of those distortions.

We find that optimal monetary policy should lean more aggressively against inflation after large cost-push shocks when the frequency of price changes is endogenously high than either after a small shock or in a quasi-linear fixed-frequency Calvo setting: it is optimal to "strike, while the iron is hot".[9] This nonlinearity establishes a key difference between our model and a standard Calvo model. Our calibration implies that this new policy prescription is relevant for the 2022-2023 inflation surge: already for inflation and frequency values of the magnitude observed during this period, optimal policy requires a significantly more aggressive anti-inflationary stance than for a small shock or under Calvo pricing.

What explains the modified policy prescription? To gain intuition about this result, we introduce a simplified model. In the simplified model, we introduce a sub-period of night, when only the firms are awake, and when the prices are fully flexible. The assumption improves the tractability of the model by turning the dynamic problem of the firms into a series of static ones, but it keeps the key underlying channel active: the repricing rate responds endogenously to aggregate shocks.

In the simplified model, both welfare and the planner's choice set can be expressed in the space of (i) the output gap, which measures the distance between output and its

[5]See, for example, Nakamura and Steinsson (2008).

[6]See, for example, Montag and Villar (2023).

[7]The Calvo model is recalibrated to generate the same price-flexibility as our baseline model for small shocks (Auclert et al. 2024). This recalibration compensates for the endogenous "selection" of large price changes, which substantially raises the flexibility of the aggregate price level.

[8]As is standard in optimal monetary policy analysis, we offset steady-state average markup distortion due to the market power with suitable subsidies. We reintroduce steady-state markup distortions only to analyze the impact of time-inconsistency.

[9]We analyze timeless Ramsey policy á la Woodford (2003).

efficient level, and (ii) inflation, as is conventional in optimal monetary policy analysis under the Calvo price setting. Welfare is dependent on these two variables, because, as we show, the output gap is closely related to the average markup and inflation is closely related to the markup dispersion and the resource costs of price changes. The choice set can also be expressed in the space of inflation and output gap – and takes the form of a nonlinear Phillips curve. It is nonlinear because, as large shocks raise the frequency of price changes and, thereby, increase price flexibility, inflation becomes more sensitive to changes in the output gap.[10]

Optimal policy leans more aggressively against inflation after large cost-push shocks than after small shocks in the simplified model, just as in the full model. In the simplified model, the relationship between the output gap and inflation under optimal policy can be illustrated by a structural "target rule." The optimal "strike while the iron is hot" policy translates into a nonlinear target rule: larger output gaps are associated with *relatively* lower inflation rates than smaller output gaps. This is a stark contrast to the corresponding target rule in the Calvo framework, which is almost linear (even without linearization).

To understand the key driving forces behind the policy prescription, it is instructive to start with the question of why the target rule is almost linear in the Calvo model. There, welfare can be well approximated by a *quadratic* function of the output gap and the inflation gap with a fixed weight.[11] Under a near-linear Phillips curve, which characterizes the Calvo framework, the Ramsey optimal policy, which can be derived as the first-order condition of the constrained maximization of the quadratic welfare subject to the Phillips curve, necessarily generates a near-linear target-rule relationship.

Why is the same relationship between inflation and output gap nonlinear under state-dependent pricing? We find that this is almost exclusively driven by the nonlinear trade-off between inflation and output gap – the nonlinear Phillips curve. Intuitively, reducing inflation in this framework is cheaper after large shocks, when the frequency is higher and the price level is more flexible – when the sacrifice ratio is low. To show that this is the dominant driving force, we combine the nonlinear Phillips curve of our simplified framework with a counterfactual quadratic welfare function approximating the Calvo model and derive a counterfactual target rule. We show that the ensuing target rule is close to the true target rule and is similarly characterized by the strike while the iron is hot policy. As the relative welfare weight of inflation and output gap is independent of the size of the shock in this counterfactual by construction, the results here are clearly driven by the state-dependent costs between inflation and output gap – the nonlinear sacrifice ratio. [12] This result generalizes to the full model. Therefore,

[10]Other papers point to complementary reasons why the Phillips curve can be nonlinear, such as state-dependent wage rigidity (Benigno and Eggertsson 2023) or Kimball aggregators (Erceg et al. 2024).

[11]This observation generalizes to large shocks the result of Woodford (2003), which shows that in the presence of small shocks, i.e. up to a second order in the neighborhood of an efficient steady state, that relevant welfare can be approximated by a quadratic function of output gap and inflation with relative welfare weights determined by structural parameters of the model (Woodford 2003).

[12]The deviation of the true welfare from the quadratic approximation actually somewhat *mitigates* the

we conclude that the key driving force behind the aggressive anti-inflationary stance after large shocks is the lower sacrifice ratio.

We establish a series of additional results in the full model. First, we show that the model features a slightly positive Ramsey optimal steady-state inflation rate, at around 0.07% per annum. This contrasts with the Calvo model, where optimal inflation is exactly zero. In our menu cost model, slightly positive steady state inflation reduces the frequency and thus helps firms to economize on costly price adjustments. In particular, it counterbalances the impact of too frequent price increases relative to price decreases, which is a consequence of the asymmetry of the profit function: firms dislike more negative price misalignments when the demand for their product is high, relative to positive misalignments when the demand is low. Second, we also find that for small cost-push shocks, optimal policy "leans against the wind": the central bank temporarily drives output below its efficient level to contain the inflationary impact of a positive cost-push shock. This is very similar, though not identical to the Calvo model, the reason, however, is different. In the Calvo model, the key distortion caused by inflation is the markup dispersion, while in our baseline model it is the resource costs due to menu costs. Third, we show analytically that the optimal response to TFP shocks is characterized by the "divine coincidence" (Blanchard and Galí 2007). In other words, optimal policy stabilizes both inflation and the output gap. Finally, we show that the well-known time inconsistency problem of monetary policy is also present in our menu cost model, although it is attenuated relative to Calvo. In both models, when the steady state is inefficient, monetary policy has the incentive to stimulate output via an unexpectedly easy policy (Galí 2008). However, in the menu cost model, such a policy is less effective on output and more inflationary because the ensuing increase in the repricing rate raises the flexibility of the aggregate price level. The time-inconsistent motive to ease is thus considerably weaker.

Our results are robust to alternative parameterizations, and also hold in the "Calvo-Plus" model (Nakamura and Steinsson 2010). The latter framework assumes that firms face a free price adjustment option with some exogenous probability. This model can better match the fraction of small price changes in the data (see also Midrigan 2011; Alvarez et al. 2021), and achieve a realistic degree of monetary non-neutrality for small shocks. However, as we show, the optimal policy implications of the model remain very close to those in our baseline.

*Related literature.* Our paper builds on the seminal article by Golosov and Lucas (2007). They propose a menu cost model (Barro 1972; Sheshinski and Weiss 1977; Caballero and Engel 1993) that has become the backbone of a positive literature studying the relationship between monetary non-neutrality and the distribution of price changes at the micro level (Midrigan 2011; Costain and Nakov 2011; Alvarez et al. 2016), as well as the impact of large aggregate shocks on inflation and activity (Karadi and Reiff 2019; Alexan-

nonlinearity of the true target rule, but its impact is quantitatively small.

drov 2020a; Auer et al. 2021). The model describes well firms' price-setting behavior in diverse environments with both low and high inflation (Nakamura and Steinsson 2008; Gagnon 2009; Alvarez et al. 2019; Gagliardone et al. 2025). This price-setting framework provides a microfounded state-dependent alternative to the canonical time-dependent Calvo (1983) model, with widely different implications in terms of both the extent of monetary non-neutrality and price flexibility as a response to large shocks. Indeed, most familiar price-setting models, such as the random menu cost model of Dotsey et al. (1999); Alvarez et al. (2021), the Calvo-plus model of Nakamura and Steinsson (2010), the rational inattention model by Woodford (2009), or the control cost model by Costain and Nakov (2019), lie on a spectrum bracketed by these two polar cases. Normative results from the Golosov and Lucas (2007) model can provide qualitative insights that generalize to a wide class of price-setting frameworks.

To the best of our knowledge, our paper is the first to solve for optimal monetary policy in this canonical menu cost model. Its main distinctive feature, relative to the textbook analysis based on Calvo (1983), such as in Woodford (2003) and Galí (2008), is a state-dependent relationship between inflation and the output gap, which has received new empirical support following the recent inflation surge (Benigno and Eggertsson 2023; Cerrato and Gitti 2023; Blanco et al. 2024a,b). Our conclusion prescribing aggressive anti-inflationary policy after large shocks is a direct consequence of this nonlinearity, which implies a favorable inflation-output trade-off that optimal policy should exploit.

Solving dynamic optimal policy in response to aggregate shocks in this framework complements previous research on optimal monetary policy, which has restricted attention to menu cost settings with a representative firm and small aggregate shocks (Nakov and Thomas 2014), sector-specific productivity shocks (Caratelli and Halperin 2023) or to optimal *steady-state* inflation rate (Adam and Weber 2019; Blanco 2021; Nakov and Thomas 2014).[13]

This paper proposes a new algorithm to solve Ramsey optimal policy in heterogeneous-agent models, building on González et al. (2024). The algorithm (i) makes the infinite-dimensional planner's problem finite-dimensional by approximating the infinite-dimensional value and distribution functions by piece-wise linear functions on a grid; (ii) accounts for the discrete price-adjustment choice using an endogenous grid; (iii) derives the FOCs of the planner's problem by symbolic differentiation; and (iv) solves the resulting set of equilibrium conditions nonlinearly under perfect foresight over the sequence space. Our approach complements other methods to solve for Ramsey policy in heterogeneous-agent models (Bhandari et al. 2021; Le Grand et al. 2022; Dávila and Schaab 2022; Nuño and Thomas 2022; Smirnov 2022).

[13] Nakov and Thomas (2014) find no significant difference between Calvo and a random menu cost model. (Caratelli and Halperin 2023) show that, in the face of sector-specific shocks, optimal policy can be characterized as *nominal wage targeting*.

## 2. Model

In the baseline economy a representative household consumes a basket of differentiated goods and supplies labor; monopolistic firms produce using a technology that is hit by both aggregate and idiosyncratic shocks and must pay a fixed menu cost to change prices; and a central bank sets interest rates. We contrast the results in our baseline economy to a Calvo economy, which is identical to our baseline except firms adjust their prices with an exogenous probability.

### 2.1. Households

A representative household consumes $C_t$, supplies working hours $N_t$ and saves in one-period, nominal bonds $B_t$, which are in zero net supply. The household maximizes

$$\max_{C_t, N_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t), \tag{1}$$

subject to

$$P_t C_t + Q_t B_t + T_t = B_{t-1} + W_t N_t + D_t, \tag{2}$$

where $T_t$ are lump-sum taxes, $W_t$ is the nominal wage, $D_t$ are lump-sum dividends from firms, and $Q_t$ is the price of the nominal bond. Aggregate consumption $C_t$ is

$$C_t = \left\{ \int_0^1 [A_t(j) C_t(j)]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where $C_t(j)$ is the quantity of product $j \in [0,1]$ and $A_t(j)$ is the idiosyncratic quality of product $j$, which follows a random walk in logs with volatility $\sigma$:

$$\log A_t(j) = \log A_{t-1}(j) + \sigma \varepsilon_t(j), \tag{4}$$

$\varepsilon_t$ is an i.i.d Gaussian innovation and $\sigma$ is a parameter.

The demand for product $j$ is

$$C_t(j) = A_t(j)^{\epsilon-1} \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} C_t \tag{5}$$

and the aggregate price index is

$$P_t = \left[ \int_0^1 \left( \frac{P_t(j)}{A_t(j)} \right)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \tag{6}$$

We assume separable utility of the CRRA class, $u(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \upsilon N_t$. Thus, equilibrium in the labor market requires:

$$w_t = \upsilon C_t^{\gamma}, \tag{7}$$

where $w_t = W_t/P_t$ is the real wage. The Euler equation is

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}e^{i_t-\pi_{t+1}}\right], \tag{8}$$

where $i_t$ is the nominal interest rate, and the the stochastic discount factor is

$$\Lambda_{t,t+1} \equiv \beta\frac{u'(C_{t+1})}{u'(C_t)}. \tag{9}$$

## 2.2. Monopolistic producers

Good $j \in [0,1]$ is produced by firm $j$ according to a constant-returns to scale technology

$$Y_t(j) = A_t\frac{N_t(j)}{A_t(j)}, \tag{10}$$

where $N_t(j)$ is labor hours, $A_t$ is aggregate productivity and $A_t(j)$ is idiosyncratic quality. Firm $j$'s nominal profit function is

$$\begin{aligned} D_t(j) =& P_t(j)Y_t(j) - (1-\tau_t)W_tN_t(j) \\ =& P_t(j)^{1-\epsilon}A_t(j)^{\epsilon-1}\left(\frac{1}{P_t}\right)^{-\epsilon}C_t - (1-\tau_t)\frac{W_t}{A_t}A_t(j)^{\epsilon}\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}C_t \end{aligned} \tag{11}$$

where $Y_t(j) = C_t(j)$ and $\tau_t$ is an employment subsidy financed by lump-sum taxes.

Crucially firm $j$ must incur a fixed “menu cost" $\eta$ labor units to change its price. This is the source of endogenenity of price stickiness in the model.

It is useful to express the firms' optimal pricing decision as a function of the price gap $x_t(j) \equiv p_t(j) - p_t^*(j)$, which is the log distance between current and optimal quality-adjusted relative price of good $j$ $p_t(j)$, where $p_t(j) \equiv \log\left(\frac{P_t(j)}{A_t(j)P_t}\right)$. The price gap is the only firm-level state variable that the pricing decision depends upon. We postulate that firms' optimal pricing follows a $(S_t, s_t)$ rule such that a firm $j$ keeps its nominal price $P_t(j)$ constant if $x_t(j) \in [s_t, S_t]$, and resets it to $x_t(j) = 0$ otherwise. Subindex $t$ subsumes all aggregate states. Idiosyncratic quality generates heterogeneity across nominal reset prices $P_t^*(j)$; however, quality-adjusted relative reset prices $p_t^*(j)$ are all identical. Thus, drop subindex $j$ to simplify notation. When a firm keeps its nominal price constant, its price gap evolves according to

$$x_t = x_{t-1} + p_t - p_t^* + p_{t-1}^* = x_{t-1} - \sigma_t\varepsilon_t - \pi_t^* \tag{12}$$

where $\pi_t^* \equiv p_t^* - p_{t-1}^* + \pi_t$ is the inflation of the quality-adjusted relative optimal reset price.

The optimality conditions for the pricing rule requires

$$V_t'(0) = 0. \tag{13}$$

$$V_t(0) - \eta w_t = V_t(s_t), \tag{14}$$

$$V_t(0) - \eta w_t = V_t(S_t), \tag{15}$$

where firms' value function $V_t(\cdot)$ is expressed only in terms of price gaps as all other states are aggregate and are subsumed by a time subindex. Equation (13) requires that the value function is maximized at the optimal reset price ($x = 0$), and equations (14,15) require indifference between resetting the price and paying the menu cost versus keeping prices constant at the endogenous ss thresholds. The value function equals

$$\begin{aligned} V_t(x) =& \Pi(x, p_t^*, w_t, A_t) \\ &+ \frac{\Lambda_{t,t+1}}{\sigma} \int_{s_{t+1}}^{S_{t+1}} \left[ V_{t+1}(x') \phi \left( \frac{x - x' - \pi_{t+1}^*}{\sigma} \right) \right] dx' \\ &+ \Lambda_{t,t+1} \left( 1 - \frac{1}{\sigma} \int_{s_{t+1}}^{S_{t+1}} \left[ \phi \left( \frac{x - x' - \pi_{t+1}^*}{\sigma} \right) \right] dx' \right) \left[ \left( V_{t+1}(0) - \eta w_{t+1} \right) \right], \end{aligned} \tag{16}$$

and comprises the current period real profits $\Pi_t(\cdot)$ and the discounted continuation value evaluated when the nominal price does not change and $x' = x - \sigma \varepsilon' - \pi'^*$ with density $\phi(\varepsilon')$ within the inaction threshold $[s_t, S_t]$, and when the firm sets a new price $x' = 0$ when its price is outside of the inaction threshold after paying the menu cost $\eta w_{t+1}$. $\phi(\cdot)$ denotes the standard normal density function. Firms' real profits are

$$\Pi_t(x, p_t^*, w_t, A_t) \equiv \frac{D_t}{P_t} = C_t \left( \exp\left( x + p_t^* \right) \right)^{1-\epsilon} - C_t(1 - \tau_t) \frac{w_t}{A_t} \left( \exp\left( x + p_t^* \right) \right)^{-\epsilon}.$$

Finally, Appendix A shows that $V_t'(0)$ can be expressed as the sum of the marginal effect of $x$ on current profits and on the expected continuation value:

$$\begin{aligned} V_t'(0) =& \Pi_t'(0) + \frac{\Lambda_{t,t+1}}{\sigma} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \left. \frac{\partial \phi \left( \frac{x - x' - \pi_{t+1}^*}{\sigma} \right)}{\partial x} \right|_{x=0} dx' \\ &+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left[ \phi \left( \frac{-S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}} \right) - \phi \left( \frac{-s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}} \right) \right] \left( V_{t+1}(0) - \eta w_{t+1} \right). \end{aligned}$$

### 2.3. Aggregation and equilibrium conditions

Firms' individual price-setting decisions give rise to an endogenous distribution price gaps, $g_t^c(x)$, which evolves according to the following law of motion

$$g_t^c(x) = \frac{1}{\sigma_t} \int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1}) \phi\left(\frac{x_{-1} - x - \pi_t^*}{\sigma_t}\right) dx_{-1} + g_{t-1}^0 \phi\left(\frac{-x - \pi_t^*}{\sigma_t}\right) \tag{17}$$

where the first term on its right-hand side describes the evolution of the density of price gaps of firms that kept their nominal prices unchanged last period while the second term is the distribution of current price gaps of firms that did change their prices last period. The frequency of price changes $g_t^0$ is

$$g_t^0 = 1 - \int_{s_t}^{S_t} g_t^c(x) dx. \tag{18}$$

Thus, the aggregate price index implies

$$1 = \int_{s_t}^{S_t} e^{(x+p_t^*)(1-\epsilon)} g_t^c(x)\, dx + g_t^0 e^{p_t^*(1-\epsilon)} \tag{19}$$

In turn, the labor-market clearing condition is given by

$$N_t = \underbrace{\frac{Y_t}{A_t}\left(\int_{s_t}^{S_t} e^{-\epsilon(x+p_t^*)} g_t^c(x)\, dx + g_t^0 e^{(-\epsilon p_t^*)}\right)}_{\text{price dispersion}} + \underbrace{\eta g_t^0}_{\text{price adjustment cost}} \tag{20}$$

such that the total hours worked equals the total use of labor for the production of aggregate output (the first term on the right-hand side), where we have imposed the equilibrium condition $C_t = Y_t$ and the loss in aggregate output due to price dispersion (the term in parenthesis), and the total amount of working hours allocated to price adjustment (the second term).

### 2.4. Aggregate Shocks

The logarithm of aggregate productivity follows a first-order autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}, \tag{21}$$

where $\rho_A \in [0, 1]$ and $\varepsilon_{A,t}$ is a mean zero aggregate productivity shock. The employment subsidy $\tau_t$ follows the autoregressive process

$$\tau_t - \tau = \rho_\tau(\tau_{t-1} - \tau) + \varepsilon_{\tau,t}, \tag{22}$$

where $\rho_\tau \in [0, 1]$, $\tau$ is the steady-state employment subsidy, and $\varepsilon_{\tau,t}$ is a mean zero cost-push shock.

### 2.5. Auxiliary models

We briefly present three alternative models we use in our analysis.

*Simplified model.* We also present a simplified version of the model to foster intuition. The simplification divides each period $t$ into two: a night and a day. The focus is the day, when all actors are awake: this is when aggregate and idiosyncratic shocks hit, households make consumption and labor supply decisions, firms reset their prices subject to adjustment cost, central bank sets interest rates and all markets clear. At night, only firms are awake, and they can reset their prices for free.

Under this setup, first, the law of motion of the price gap distribution collapses every night by construction as each firm closes its price gap ($x_{-1}(j) = 0$ for each $j$). Second, when making a decision during the day, firms set prices only with the current period in mind, as if their discount rate were zero ($\beta = 0$). They do this because they know that they will be able to reset their prices for free again the next night.

The simplification maintains the state-dependent nature of the firms' price-setting problem: both how many and which prices change will be decided endogenously as a response to the aggregate and idiosyncratic shocks, taking into account the conduct of monetary policy. We gain tractability, however, by replacing the dynamic problem of the firms with a series of static problems, where future expectations play no role in the price-setting decisions. The advantage of the approach is that, as we show below, objects familiar from conventional optimal policy analysis like the Phillips curve, which describes the trade-off between inflation and output gap, and the target rule, which describes the relationship between inflation and output gap under optimal policy, become structural. Despite its simplicity, the simple model generates results that are not only qualitatively but also quantitatively similar to analogous objects in the full model.

The model economy can be described by seven equations. First, firms' log quality-adjusted relative reset price ($p^*$) is set at a constant markup over its marginal costs

$$e^{p^*} = \frac{\epsilon}{(\epsilon - 1)}(1 - \tau)w. \tag{23}$$

Second and third, the firms' price adjustment decision is characterized by the threshold values which equate current profits under unchanged price $p$, $\Pi(p)$, with profits under the optimal price $p^*$ reduced by the menu costs, $\Pi\left(e^{p^*}\right) - \eta w$. Using equation (23 and Descartes' Rule of Signs, there are exactly 2 positive real roots, $s < S$.

$$\left((e^{p^*})^{1-\epsilon} - (1-\tau)w(e^{p^*})^{-\epsilon}\right) - \eta = \left(s^{1-\varepsilon} - (1-\tau)ws^{-\epsilon}\right), \tag{24}$$

$$\left((e^{p^*})^{1-\epsilon} - (1-\tau)w(e^{p^*})^{-\epsilon}\right) - \eta = \left(S^{1-\varepsilon} - (1-\tau)wS^{-\epsilon}\right), \tag{25}$$

The remaining 4 equations are as in the full model.[14] They are given by the ana-

[14] The equations are listed in Appendix F.

logues of the households' labor supply (7), the frequency of price changes (18), the labor-market-clearing condition (20) and the definition of the price level (19). Free price changes in the preceding night implies that $g^c_{-1}(x) = 0$, $g^0_{-1} = 1$, $p^*_{-1} = 1$) and, therefore, the price distribution under normally distributed idiosyncratic shocks and inflation rate $\pi$ is now normal with mean $\pi$ and variance $\sigma^2$: $g^c(p) = \phi\left(\frac{p+\pi}{\sigma}\right)$. These seven equations define an equilibrium in 8 variables $w, \pi, C, N, s, S, g^0, p^*$. The policy maker has one degree of freedom to choose $\pi$.

*Nonlinear Calvo model.* This model is a natural benchmark that we contrast our model to. It is identical to the baseline economy without idiosyncratic shocks $\sigma = 0$ or menu costs $\eta = 0$. Instead, firms face an exogenous price change probability $\theta$ as in Calvo (1983).

*Calvo-plus model.* We use this model to study the robustness of our main results. It is an extension of our baseline model proposed by Nakamura and Steinsson (2010), where the menu cost is stochastic: it equals $\eta$ with probability $\alpha$ and zero otherwise. The extension improves the realism of the framework both by better capturing the distribution of price changes through the introduction of small price changes, and better matching the extent of monetary non-neutrality obtained by time-series evidence. Appendix G describes the details.

## 3. Optimal monetary policy problem and computational approach

We start our analysis by introducing the central bank's problem. We consider the Ramsey problem, i.e., optimal monetary policy under commitment. We also present a new computational method to deal with the complexities associated with the problem's high dimensionality, and we specify our baseline calibration.

### 3.1. Ramsey problem

The central bank selects the paths for all equilibrium variables subject to the competitive equilibrium conditions. Combining households' utility function in (1) and the market-clearing conditions for final output $C_t = Y_t$ and for labor (20), the problem of a benevolent central bank is

$$\max_{\substack{\{w_t, Y_t, V_t(\cdot), S_t, s_t,\\ p^*_t, \pi^*_t, g^c_t(\cdot), g^0_t\}_{t=0}^{\infty}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{Y_t^{1-\gamma}}{1-\gamma} - \upsilon \frac{Y_t}{A_t} \left( \int_{s_t}^{S_t} e^{-\epsilon(x+p^*_t)} g^c_t(x)\, dx + g^0_t e^{-\epsilon p^*_t} \right) - \upsilon\eta g^0_t \right)$$

subject to the labor supply (7), firms' value function $V_t(\cdot)$ (16), firms' optimal pricing $\{s_t, S_t, p^*_t\}$ (13), (14), and (15), the definition for inflation in quality-adjusted relative optimal reset price $\pi^*_t$, the distribution of price gaps $g^c_t(\cdot)$ (17), the frequency of price changes $g_t^0$ (18) and the aggregate price index (19). Additional constraints are the stochastic processes for aggregate productivity $A_t$ (21), and cost-push shocks $\tau_t$ (22).

Two observations are due. First, we follow the approach in standard optimal monetary analysis (Woodford 2003; Galí 2008) of separating the Ramsey problem in two: the equilibrium pinned down by a benevolent central bank and the implementation problem, i.e. the nominal interest rates path consistent with the equilibrium according to the household's Euler equation (8).

Second, note that the constraints set for this problem are continuous and differentiable despite the individual firm's price policy function is not. This is so because each firm has zero mass, and thus the discontinuity in a single firm's behavior does not lead to a discontinuity in aggregates. Furthermore, $V_t(x)$ and $g_t^c(x)$ are continuously differentiable over the relevant range $(s_t, S_t)$.

### 3.2. Computational solution method

We solve the problem with a new algorithm in discrete time, which is similar to that in González et al. (2024). The core idea is to represent the problem as a high-dimensional dynamic programming problem in which the Bellman equation and the law of motion of the price-gap distribution are constraints.

The solution of this Ramsey problem is involved because the value function $V_t(\cdot)$ and the distribution $g_t^c(\cdot)$ are infinite-dimensional variables. This poses a challenge as we need to compute the first-order conditions (FOCs) with respect to these infinite-dimensional variables.[15] Our algorithm first discretizes the planner's objective and constraints and then determines the FOCs, instead of first determining the FOCs for the planner's continuous space problem, and then discretizing them. Thus, we transform the original infinite-dimensional problem into a high-dimensional problem, in which the value function and the state density are replaced by large vectors with a dimensionality equal to the number of grid points used to approximate the individual state space. This approximation needs to be smooth and accurate enough to capture the higher-order effects of policy.

An additional computational challenge is that a simple discrete-state approximation may fail, as equilibrium conditions include discrete choices. Therefore, we approximate the distribution and value function not by discrete functions on a predetermined

[15]There are a number of proposals in the literature to deal with this problem. Nuño and Thomas (2022), Smirnov (2022), and Dávila and Schaab (2022) deal with the full infinite-dimensional planner's problem in continuous time. This implies that the Kolmogorov forward (KF) and the Hamilton-Jacobi-Bellman (HJB) equations are constraints faced by the central bank. They derive the planner's FOCs using calculus of variations, thus expanding the original problem to also include the Lagrange multipliers, which in this case are also infinite-dimensional. These papers solve the resulting differential equation system using the upwind finite-difference method of Achdou et al. (2021). Bhandari et al. (2021) make the continuous cross-sectional distribution finite-dimensional by assuming that there are $N$ agents instead of a continuum. They then derive standard FOCs for the planner. In order to cope with the large dimensionality of their problem, they employ a perturbation technique. Le Grand et al. (2022) employ the finite-memory algorithm proposed by Ragot (2019). It requires changing the original problem such that, after $K$ periods, the state of each agent is reset. In this way the cross-sectional distribution becomes finite-dimensional.

grid, but by *piece-wise linear* functions over an *endogenous* grid. The endogenous grid is selected to always include the two boundaries of the inaction region $[s_t, S_t]$ and price gap $x_t = 0$. Furthermore, we explicitly take the mass point at 0 into account in the distribution. Integrals to compute expectations are evaluated algebraically, conditional on those piecewise linear functions.

Appendix E presents the details of the computational solution method. The Bellman equation can thus be approximated over a grid of price gaps $x$ as

$$\mathbf{V}_t = \mathbf{\Pi}_t + \left[\mathbf{A}_t \mathbf{V}_{t+1} - \mathbf{b_{t+1}} \eta w_{t+1}\right],$$

where $\mathbf{V}_t$ and $\mathbf{b}_t$ are vectors with the value function and the expected adjustment probability evaluated at different grid points, respectively, and $\mathbf{A}_t$ is a matrix that captures the idiosyncratic transitions due to firm-level quality shocks and aggregate inflation. Similarly, the law of motion of the density for $x \neq 0$ is

$$\mathbf{g}_{\boldsymbol{t}}^{\boldsymbol{c}} = \mathbf{F}_t \mathbf{g}_{\boldsymbol{t-1}}^{\boldsymbol{c}} + \mathbf{f}_t g_{t-1}^0,$$

where $\mathbf{g}_{\boldsymbol{t}}^{\boldsymbol{c}}$ and $\mathbf{f}_t$ are vectors representing the probability distribution function and the scaled and shifted normal distribution, respectively, $\mathbf{F}_t$ is a matrix that captures the evolution of the price distribution due to firm-level quality shocks and aggregate inflation. We define

$$g_t^0 = 1 - \mathbf{e}_t^\top \mathbf{g}_{\boldsymbol{t}}^{\boldsymbol{c}}.$$

as the mass point at $x_t = 0$ where $\mathbf{e}_t$ is a vector of weights corresponding to the trapezoid rule. The labor market clearing condition and the aggregate price index can be written in a similar form. Once we have the discretized version of the problem, we find the planner's FOCs by symbolic differentiation. This delivers a large-dimensional system of difference equations, as we have Lagrange multipliers associated with each grid point of the value function or the probability function.

Next, we find the Ramsey steady state. To do so, we construct a nonlinear multi-dimensional function mapping inflation to the rest of the steady-state equilibrium variables. We then combine this function with the planner's FOCs. As the system is linear in Lagrange multipliers, the solution boils down to finding the zero of a non-linear function of the initial variable (inflation) using the Newton method. Finally, to compute the dynamics of the Ramsey problem, we solve the system of difference equations non-linearly in the sequence space also using the Newton method.

The symbolic differentiation and the two applications of the Newton method can be automated using several available software packages, in our case, Dynare (Adjemian et al. 2023). The approach is compatible with the nonlinear sequence-space Jacobian toolboxes. This algorithm can be employed to compute optimal policies in a large class of heterogeneous-agent models. Compared to other algorithms, it stands out as easy to implement. González et al. (2024) show that this algorithm delivers the same results as computing the FOCs by hand using calculus of variations and then discretizing the

|  | $\beta$ | $\epsilon$ | $\gamma$ | $\upsilon$ | $\eta$ | $\sigma$ | $\tau$ | $\rho_A$ | $\rho_\tau$ |
|---|---|---|---|---|---|---|---|---|---|
| Baseline | $0.96^{1/12}$ | 7 | 1 | 1 | 0.010 | 0.012 | 0.1435 | $0.95^{1/3}$ | $0.9^{1/3}$ |
| Simplified model | $0.96^{1/12}$ | 7 | 1 | 1 | 0.004 | 0.021 | 0.1431 | 0 | 0 |

TABLE 1. Parameter values

model. Our algorithm runs in a few minutes on a normal laptop.

### 3.3. Calibration

Table 1 presents the calibration of our baseline and the simplified model. We set the discount factor to $0.96^{1/12}$, which implies a steady-state real interest rate of 4%. The elasticity of substitution across products is $\epsilon = 7$, as in (Golosov and Lucas 2007). We assume log utility in consumption ($\gamma = 1$), as in Midrigan (2011), and we set the utility weight of leisure to unity ($\upsilon = 1$).

We calibrate the menu cost and the standard deviation of idiosyncratic shocks to match two target moments: an 8.7% frequency of price changes in the steady state as documented for the U.S. in Nakamura and Steinsson (2008), and a 20% frequency at 10% inflation rate broadly in line with the peak values observed in the U.S. in 2022 as documented by Montag and Villar (2023). In the baseline model, the implied menu cost is $\eta = 1\%$ and the steady-state standard deviation of idiosyncratic quality shocks is $\sigma = 1.2\%$. The steady-state labor subsidy $\tau$ is set to ensure that output is at its efficient level.

Finally, the persistence of shocks is taken from Smets and Wouters (2007), once transformed from quarterly to monthly frequency: $\rho_A = 0.95^{1/3}$ for aggregate productivity shocks and $\rho_\tau = 0.9^{1/3}$ for employment subsidy shocks (interpreted as cost-push shocks).

These parameters are inherited also by the additional auxiliary models. The same calibration targets imply an $\eta = 0.4\%$ and $\sigma = 2.1\%$ in the simple model. In the Calvo model, we disregard idiosyncratic shocks $\sigma = 0$ and calibrate the probability of price adjustment $(1 - \theta)$ so as to make the Calvo model imply an identical response to a small monetary policy shock than our baseline model (Auclert et al. 2024). This requires a parameter $\theta = 40\%$.

## 4. Strike while the iron is hot

This section focuses on our main result: the nonlinearity of optimal monetary policy as a response to cost-push shocks.[16] We first present numerical simulations in our full model to characterize the nature of the non-linearity and to contrast it with the conventional Calvo framework. Then we describe its main driving forces relying on

[16] We defer to Section 5 for an analysis of the steady state of the optimal Ramsey problem.

the simplified model. We close the section by showing robustness in the CalvoPlus model.

The section analyzes *timeless* optimal monetary policy (Woodford 2003; Galí 2008). This corresponds to the optimal monetary policy starting from the Ramsey steady state, and all of the Lagrange multipliers are initialized at their steady-state values.[17]

### 4.1. Nonlinear optimal monetary response to cost-push shocks

How should optimal monetary policy react to cost-push shocks of different sizes, and how do reactions in our baseline model compare to those in the canonical Calvo framework? Figure 1 shows impulse responses to a large cost-push shock ($\varepsilon_{\tau,t}$, blue solid line) in the baseline model, and contrasts it to linearly-scaled impulse responses to a small cost-push shock (yellow dotted line)[18]; and to a large cost-push shock in the Calvo model (red dashed line). The size of the large shock is calibrated to generate a 20% frequency at the peak in the baseline model, a 12.3 percentage point increase from the 8.7% frequency at the steady state. The magnitude of the frequency increase is broadly in line with that observed during the 2022-2023 inflation surge in the U.S. (Montag and Villar 2023).[19]

The optimal policy response "leans against the wind" in all three cases. The central bank tolerates an inflation increase (panel a) to partially cushion the decline in output (panel b).[20] Optimal policy implies a temporary decline in the real interest rate in parallel with the spiking inflation, but prescribes a commitment to a persistently tight policy stance in the future.

Optimal monetary policy in the menu cost model is nonlinear. Impulse responses to the large shock are significantly different from the linearly-scaled responses to a small shock. Notably, the frequency under a large shock increases substantially, while it remains almost unchanged after the small shock (panel d). This nonlinear frequency response is an inherent feature of the model. Consider a small inflationary shock. The repricing frequency stays unchanged because the fall in the frequency of price decreases almost completely offsets the rise in the frequency of price increases. If one considers instead a large inflationary shock, the price decreases fade out and the rise in the frequency of price increases quickly dominates, thus producing an overall increase in the repricing frequency (Gagnon 2009; Karadi and Reiff 2019; Alvarez and Neumeyer 2020; Alexandrov 2020b; Cavallo et al. 2023).

The optimal policy is characterized by a more aggressive monetary policy (panel c)

[17] We assess the implications on the time-inconsistency of optimal policy in our state-dependent framework in Section 5.

[18] The scaling factor is the ratio between the shock size of the large (68%) and the small shock (0.25%).

[19] The impulse responses are computed nonlinearly under perfect foresight. For small shocks, this is equivalent to the first-order approximation to the stochastic problem, as discussed by Boppart et al. (2018). For large shocks, its interpretation is similar to that in Cavallo et al. (2023): an unexpected once-and-for-all large shock that hits the economy in the deterministic steady state.

[20] For cost-push shocks, output equals the output gap, as this type of shock yields no variation of efficient output.

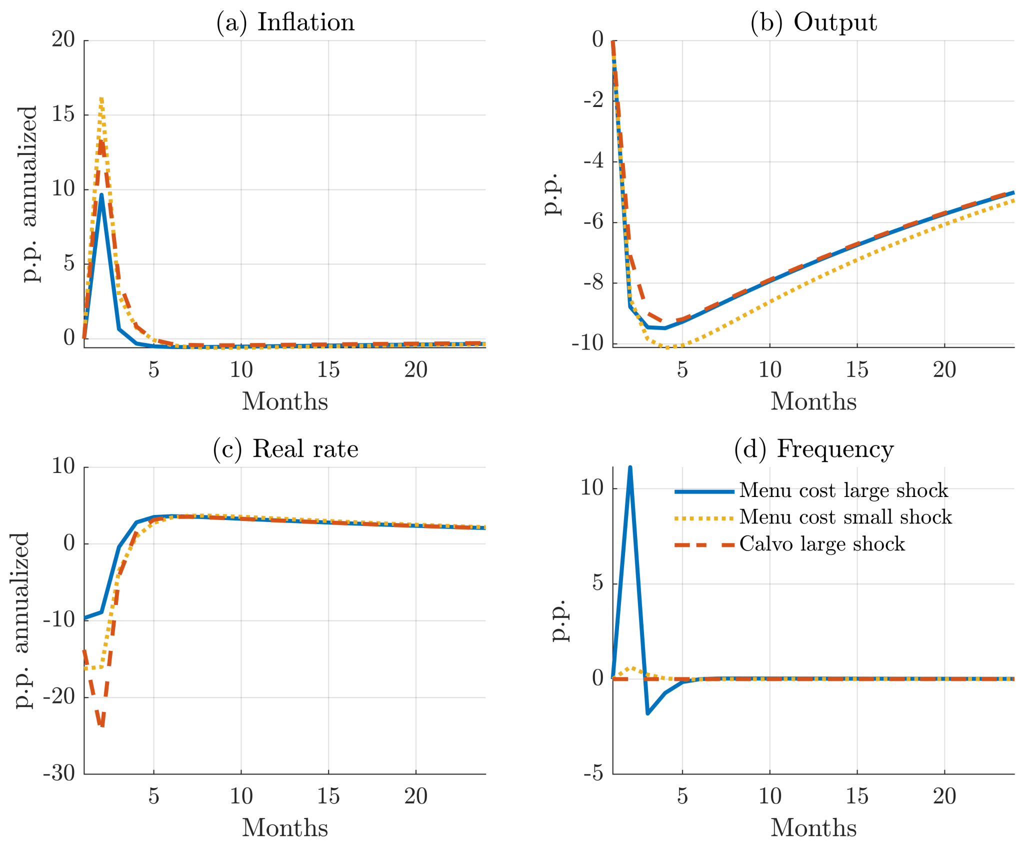


FIGURE 1. Impulse responses to a cost-push shock under the optimal monetary policy

The figure shows impulse responses to a large cost-push shock in the baseline menu cost model (blue solid line); and it contrasts the responses with those of a linearly-rescaled small cost push shock in the baseline model (yellow dotted line) and a large cost-push shock in the Calvo model (red dashed line).

after the large cost-push shock, which raises the frequency of price changes, than after the small shock. The central bank "strikes while the iron is hot". The tighter policy leads to a substantially more muted increase in inflation after the large shock than after the linearly-scaled small shock (panel a). The output effects are broadly similar (panel b).[21] The response to the large shock in the Calvo model is also broadly aligned with that of the linearly-scaled small shock in the menu cost model.

Figure 2 displays the responses of key macro variables under optimal policy for a range of different adverse cost-push shock sizes in the menu cost model (blue solid line), in the Calvo model (red dashed line), and in a counterfactual menu-cost model (yellow dashed line) described below. In particular, it draws the peak responses of inflation, output, and frequency, as well as the cumulative response of the real interest rate ($\sum_{t=1}^{24}(i_t - \pi_{t+1})$).

[21] The output effect is somewhat smaller for large adverse cost-push shocks than for linearly-scaled small shocks in the Calvo model (see Figure 2 below). This nonlinearity of the underlying framework is inherited in the menu cost model, which explains why output declines slightly more in the case of small shocks.

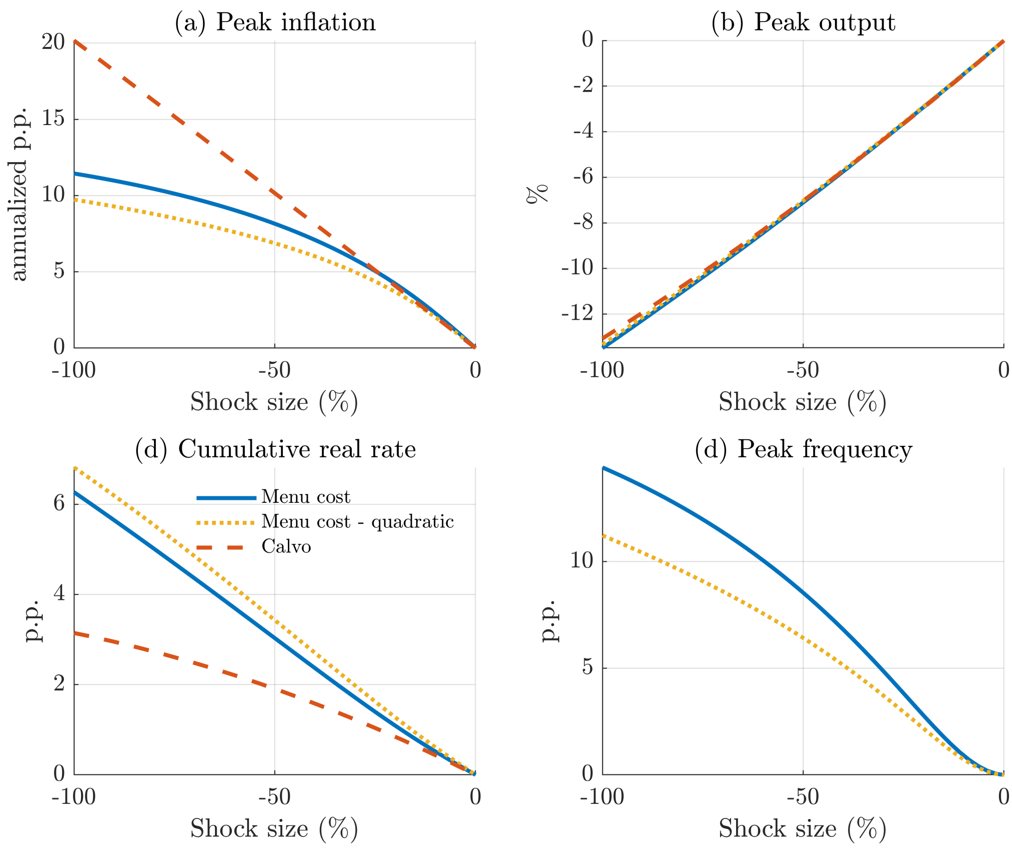


FIGURE 2. Optimal response to a cost-push shock for different shock magnitudes.

The figure displays the difference in the value of inflation, output gap, and repricing frequency between the period after the shock arrival and the value in the deterministic steady state. The real interest rate is evaluated over the first 2 years of the shock.

The peak frequency response in the baseline model has a half-bell curve shape. This confirms the nonlinear nature of the optimal frequency response outlined above: frequency stays unresponsive for small shocks, but responds strongly to large shocks. The cumulative real rate figure (panel c) confirms that the policy is more aggressive in the menu cost model for large shocks than for small shocks and than in the Calvo model (red dashed line).[22] In line with the more aggressive policy in the menu cost model, the peak output effect is somewhat larger than in the Calvo model (panel b). Finally, the response of inflation increases less than proportionally with shock size (solid blue line, panel a) in the menu cost model, which is in contrast with the near-linearity of the inflation response of the Calvo model (dashed red, panel a). When shocks are small, the optimal response of inflation in both models is near-linear and has a similar – though not identical – slope.

[22]Though hard to perceive visually, the slope of the solid blue line in panel c is similar to that of the dashed red line when the shock size is close to zero. The cumulative real rate as a function of shock size is thus slightly convex in the menu cost model while concave in the Calvo model.

### 4.2. Inspecting the mechanism in the simplified model

In order to provide intuition for the driving forces behind the strong anti-inflationary stance of the central bank after large cost-push shocks, we analyze the simplified version of the model introduced in Section 2.5. As outlined there, the simplification introduces a subperiod of night, when the firms can reset their prices for free. This simplification transforms the dynamic problem into a series of static problems, but keeps the key channel active: firms decide endogenously when to adjust prices in response to the aggregate shock and to monetary policy.

We cast the planner's problem as a 2-dimensional optimization problem in output[23] and inflation, in analogy to the well-known textbook analysis of optimal monetary policy. A key advantage of the simplified model is that we can do this without any need for approximation. We first inspect the planner's choice set, defined by the Phillips curve, and then his objective.

*Phillips curve.* We can summarize the private equilibrium in this economy by a single equation in inflation and output by combining the first 6 equations[24] to eliminate $w, s, S, g^0, p^*$:

$$1 = \left[ \int_{\mathrm{s}}^{\mathrm{S}} e^{(p)(1-\epsilon)} \phi\left(\frac{p+\pi}{\sigma}\right) dp + \left(\frac{\varepsilon\,(1-\tau)}{\varepsilon-1} Y\right)^{1-\epsilon} \left[1 - \int_{\mathrm{s}}^{\mathrm{S}} g^c(p+\pi)\,dp\right]\right], \tag{26}$$

where s and S are implicit functions.[25] This equation implicitly defines a relationship between inflation and output (or, equivalently, the output gap): the Phillips curve.

Panel a of Figure 3 shows the Phillips curve in the calibrated model and compares it to the case of Calvo pricing. In both cases, the curves are increasing and at zero inflation the slopes are identical in the two models.[26] The key difference between Calvo and state-dependent pricing models is that while in Calvo the Phillips curve is near linear, the curve is convex in the menu cost model: that is, as inflation increases, its expansionary effect on output diminishes.

Panel (b) of Figure 3 depicts the relationship between the frequency of price changes and inflation. When inflation is low, the frequency of price changes remains close to its steady-state level, 8.7% in our calibration. Thus, locally the economy behaves similarly to a standard sticky-price economy à la Calvo (1983). However, as shocks get larger, the responses of both inflation and frequency rise. This makes aggregate

[23]The efficient output is $Y^e = 1$, so log output equals to log output gap.

[24]Equations determining the optimal quality-adjusted relative price, the inaction thresholds, the household labor supply equation, the frequency of price adjustment and the price level. We also impose $Y = C$.

[25]$\mathrm{S}(Y,\tau)$ and $\mathrm{s}(Y,\tau)$ are the two roots of the equation $\left(\frac{\epsilon}{\epsilon-1}(1-\tau)Y\right)^{1-\epsilon} - ((1-\tau)Y)^{1-\epsilon} - \eta = \mathrm{x}(Y,\tau)^{1-\epsilon} - (1-\tau)Y\mathrm{x}(Y,\tau)^{-\epsilon}$ for $\mathrm{x} = \mathrm{s}, \mathrm{S}$.

[26]This happens by construction, as the Calvo model is re-calibrated to replicate the slope of the Phillips curve in the limit as $\tau$ goes to zero.

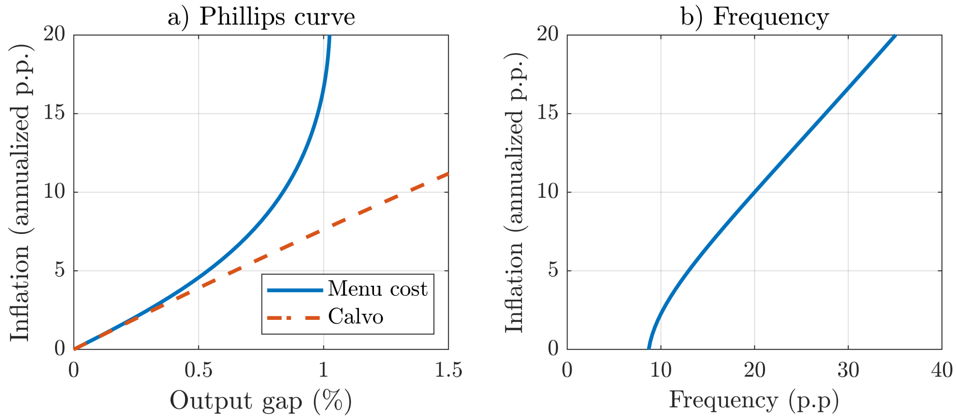


FIGURE 3. Phillips curve

Panel a plots the Phillips curve implicitly defined by equation (26), as well as the counterfactual value in the case of Calvo pricing, while panel b displays the mapping between frequency and inflation in the simplified model.

prices more flexible, which implies that the responsiveness of output decreases with inflation.[27]

The Phillips curve describes the choice set of the policy maker setting inflation. Its slope reflects the state-dependence of the inflation-output trade-off involved in monetary policy decisions: It states how much the output gap must decline to reduce inflation by a unit, also known as the *sacrifice ratio* of monetary policy. This slope more than doubles when frequency reaches 20%, a magnitude documented during the post-COVID inflation surge (Montag and Villar 2023). While in a low-frequency and low-inflation environment, the sacrifice ratio is high, it becomes much lower once frequency and inflation increase.[28]

*Welfare.* Turning next to the central bank's preferences, its objective is given by household utility $U = \log(C) - N$. Using the firms' conditions on the optimal quality-adjusted relative price and the adjustment thresholds, the definition of the price level, the frequency, and the labor-market clearing condition, we can express the utility gap, which is the difference of the utility from the utility at the efficient allocation ($Y = N = 1$), in the simple model as:

$$(27) \quad U - U^e = \underbrace{\log(Y) - (Y-1)}_{\text{Output gap}} - \underbrace{\eta\left[1 - \int_s^S \phi\left(\frac{p+\pi}{\sigma}\right) dp\right]}_{\text{Adjustment costs}} -$$

[27]At frequencies over 40%, roughly corresponding to inflation levels around 20% annualized, the Phillips curve becomes backward bending (not shown). At this level, monetary policy reach its maximum effectiveness in stimulating activity, and any further inflation reduces output. However, such levels of inflation and frequency are fairly extreme. For the rest of the analysis we restrict our attention to inflation levels that can be large, but not as large as to go beyond this point.

[28]Blanco et al. (2024b) also discuss how the sacrifice ratio changes with the level of inflation

$$Y\underbrace{\left(\int_{\mathrm{s}}^{\mathrm{S}} e^{p(-\epsilon)}\phi\left(\frac{p+\pi}{\sigma}\right)dp+\left(1-\int_{\mathrm{s}}^{\mathrm{S}}\phi\left(\frac{p+\pi}{\sigma}\right)dp\right)e^{\mathrm{p}^*(\pi)(-\epsilon)}-1\right)}_{\text{Price dispersion}}$$

where where s, S and $\mathrm{p}^*$ are implicit functions.[29] Just like the Phillips curve, this welfare function depends only on inflation and output.[30]

The utility gap can be decomposed into three terms. The first term is a nonlinear function of the log output gap $\tilde{y} = y - y^e$, where $y \equiv \log Y$. In particular, the first term on the right hand side equals $\tilde{y} - (e^{\tilde{y}} - 1)$. The second term is the adjustment cost that is the product of the menu cost and the frequency of price adjustment. The third term is the product of the output and the deviation of price dispersion from 1. In Appendix C.2, we show that the utility gap can be equivalently expressed in terms of the average markup $\overline{\mu} = \log\left(\int e^{\mu(j)(1-\epsilon)}dj\right)^{\frac{1}{1-\epsilon}}$, the adjustment cost, and the markup dispersion $\zeta^{\mu-\overline{\mu}} \equiv \int e^{(\mu(j)-\overline{\mu})(-\epsilon)}g(\mu(j)-\overline{\mu})dj$ as

$$(28)\quad U - U^{\mathrm{e}} = \underbrace{-\log(1-\tau)-\overline{\mu}-\left(\frac{1}{e^{\overline{\mu}}(1-\tau)}-1\right)}_{\text{Average markup gap}} - \underbrace{\eta g_0.}_{\text{Adjustment costs}} - Y\underbrace{\left(\zeta^{\mu(i)-\overline{\mu}}-1\right)}_{\text{Price dispersion}}$$

Appendix C.2 also shows that the output gap equals the average markup gap $\tilde{y} = -(\log(1-\tau_t) + \overline{\mu}_t)$, and that the price dispersion and the markup dispersion are equal. Therefore, welfare costs are intuitively caused by the *misallocation* caused by the deviation of firms' markups $\mu(j) = p(j) - mc(j)$ from their efficient levels.[31] Misallocation can be decomposed into the "average markup gap" caused by a nonlinear function of the tax-adjusted average markups, $\overline{\mu} + \log(1-\tau)$, and the "markup dispersion". The average markup gap thus describes *aggregate* over- or under-consumption, while the markup (price) dispersion refers to the inefficient *relative* consumption of different goods. Second, labor is inefficiently allocated to price adjustment (*menu costs*), which is captured by the second component.

In the simplified model, equation (27) shows that the average markup gap depends only on the output gap while price dispersion and adjustment costs depend only on inflation. In Calvo, the same decomposition applies, with the adjustment cost term being zero. Figure 4 illustrates this decomposition for the simplified menu cost model and contrasts it to the same decomposition in Calvo. The average markup gap coincides for the two models (panel b). Price dispersion increases in inflation in Calvo, while it mildly decreases in inflation in the menu cost model (panel c). This decrease is due to the endogenous selection of adjusters with large markups. The adjustment costs (panel d) increase with inflation in the menu cost model in line with the endogenous increase

[29] $\mathrm{s}(\pi)$, $\mathrm{S}(\pi)$ and $\mathrm{p}^*(\pi)$ solve the SS band conditions, and the definition of the price level.

[30] In the Calvo case without idiosyncratic shocks this representation of the welfare function, when approximated to second order, yields the well-known loss function $-\frac{1}{2}\left[\hat{y}^2 + \epsilon\left(\frac{1-\theta}{\theta}\right)\hat{\pi}^2\right]$ (see Galí 2008) where the 'hat' denotes deviation from the deterministic steady state.

[31] The steady-state employment subsidy is set to offset the inefficiency of steady-state markups.

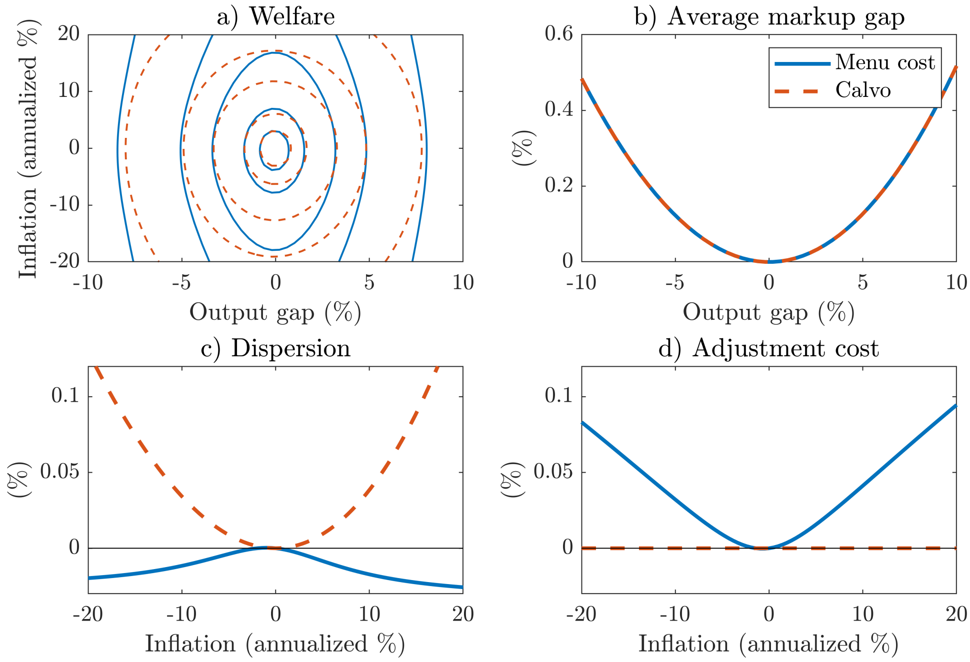


FIGURE 4. Welfare decomposition

Note: Decomposition of welfare differences according to equation (28). Welfare gaps are expressed in % of efficient consumption.

in the fraction of firms updating their prices. Summing up the two pricing frictions and comparing them to price dispersion in Calvo, we see that the welfare effects of nominal rigidities are u-shaped in inflation in both models. Quantitatively, however, the losses from nominal rigidities are somewhat smaller in the menu cost model (Burstein and Hellwig 2008). Thus, the central bank is somewhat less inflation-averse compared to Calvo, which is reflected in the different degrees of ellipticity of the iso-welfare curves shown in panel a.

*Optimal policy.* We can now set up the central bank's problem. It chooses inflation $\pi$ and output $Y$ so as to maximize the objective (27) subject to the Phillips curve (26). Figure 5 represents this problem and its solution graphically. It shows the Phillips curve (PC, dashed lines) for a particular value of the exogenous cost-push shock $\tau$; and the utility isoquant (thin solid lines) that tangents that PC. The optimal policy is defined by their tangential point, points A and B for Calvo and menu cost pricing, respectively. The target rule traces these points for different levels of the cost-push shock (thick solid line).

Three key insights can be derived from this figure. First, the slope of the target rule at zero is slightly smaller in the menu cost economy than in the Calvo economy. Since the Phillips curve slopes coincide for both models at zero by construction, the different slope of the target rule is exclusively due to the fact that the welfare function is less anti-inflationary in the menu cost model. This effect is quantitatively small.

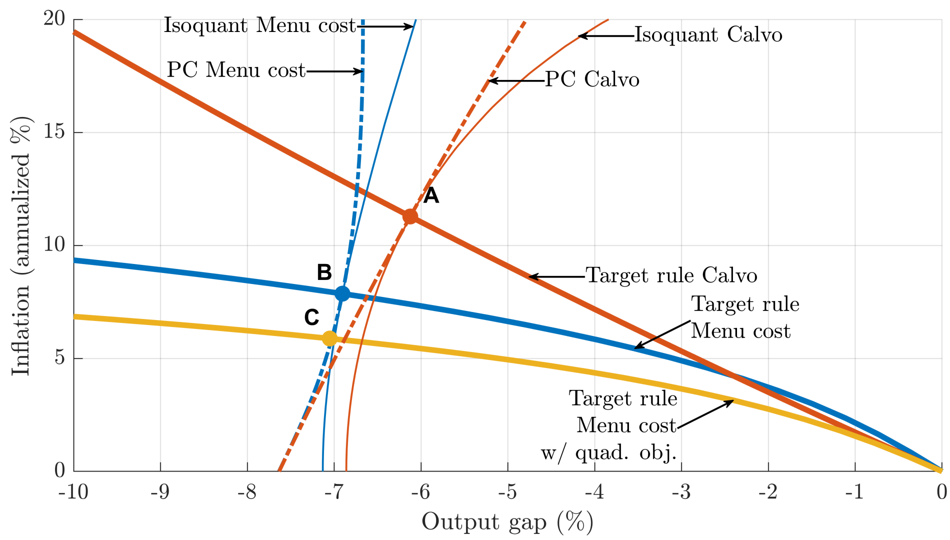


FIGURE 5. Optimal policy and the target rule

This figure combines the welfare functions from panel a in Figure 4 with the Phillips curves from panel a in Figure 3 to derive the target rule.

Second, while the target rule is almost linear under Calvo pricing (red), under menu cost pricing, the target rule is concave (blue). This implies that the central bank leans more and more aggressively against inflation as inflation increases. The central bank *strikes while the iron is hot.*

This begs the question why. Calvo and menu cost models differ in both the objective and the constraint. Which one is responsible for this? To shed light on this question, we compute optimal policy assuming that the central bank faces the Phillips curve of the menu cost framework, but behaves according to Calvo preferences.[32] That is, we look for tangential points of the blue Phillips curve with the red iso-welfare curve. Point C marks this point for the given Phillips curve. The yellow line traces these points out for all levels of the cost-push shock. Since the Calvo central bank is slightly more inflation averse, the yellow line is lower than the blue menu-cost target rule. Yet, the degree of concavity is similar. We can thus conclude that the non-linearity of the target rule in the menu cost model is driven by the strong concavity of the Phillips curve. The shape of the objective function, if anything, diminishes the concavity of the target rule a bit, relative to Calvo. This is the third insight.

What is the intuition behind striking while the iron is hot result? In the case of small shocks, the change in the frequency of price adjustments is negligible, and thus the logic of the Calvo framework still applies: The central bank tolerates some inflation to partially cushion the fall in the output gap. However, as inflation rises, frequency starts to pick up and prices become more flexible. This reduces the sacrifice ratio: to achieve the same impact on the output gap, the central bank would need to let inflation

[32] Calvo preferences are approximated to second order in both cases. However, since this approximation is very tight, it makes no relevant difference.

increase substantially more in this case, and it is not willing to do so. Thus, after a large cost-push shock, the central bank stabilizes inflation more relative to the output gap than after small shocks. The central bank "leans against frequency", tightening policy more aggressively in the case of a large shock that increases frequency. In the nonlinear Calvo model (red dashed line), by contrast, the non-linearity is negligible.

### 4.3. Relation to the full model

In the nonlinear full dynamic model, the Phillips "relationship" is a dynamic multidimensional relationship, depending both on current and expected state variables and is described by several equations. There is thus no simple structural relationship linking current inflation and output any more.[33] Instead, the Phillips relationship is made up by a dynamic block of equations which contains not only the definition of the price level (19), the firms' optimality conditions (now dynamic) (13-15) and the definition of frequency (18) as in the static model, but also the law of motion of the distribution (17) and the value function (16). A similar argument applies to the welfare function in terms of current inflation and output. The table in the Appendix B compares the equilibrium conditions of the simplified (static) Calvo and menu cost models to those of the full model.

Nevertheless, much of the intuition carries over and it is still useful to think broadly in terms of objective and constraints. To illustrate this, Figure 6 compares the Phillips relationship and the target relationship from the simplified model with the relationship between peak output and peak inflation implied by the full model after a cost push shock under optimal policy (panel a) and after a monetary policy shock under a Taylor rule (panel b). In all cases we display the response of variables to shocks of different magnitudes at the peak starting from the steady state of the Ramsey problem. As explained above, in the full model these relationships are not structural, but are conditional on the initial conditions and the shock process. Nevertheless, these two relationships are fairly stable with respect to those conditions and are surprisingly similar to the structural relationships uncovered from the simplified model.

Interpreting this figure for the full model, two features are worth noticing. First, the slope of the output-inflation relationship (the target relationship) under optimal policy at zero is almost indistinguishable from that under Calvo (panel a), which, is given by $-1/\epsilon$ up to a first order (see Galí 2008). The Calvo model thus delivers a good approximation of optimal policy for moderate levels of inflation.

Second, the nonlinearity of the menu cost model becomes quantitatively significant quite quickly. At 10% inflation for example, the slope of the Phillips relationship is 1.5 times larger than under Calvo pricing. At the same inflation level, the optimal policy response to a cost push shock is almost 50% more restrictive in terms of output. Thus, while at moderate inflation levels the Calvo model is a good enough approximation of

[33] Note that the same is true in the nonlinear Calvo model, the Phillips curve only emerges under linear approximation.

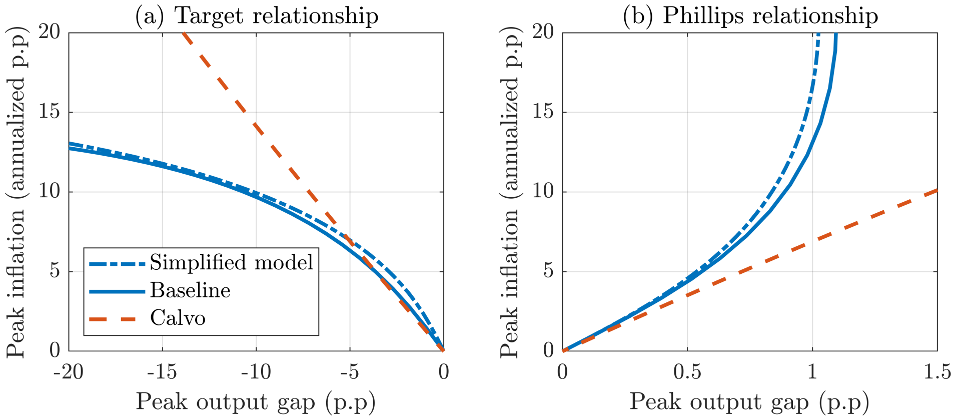


FIGURE 6. Simplified vs full model

Note: Panel a shows the response on peak inflation and output gap in the menu cost and Calvo model to a monetary policy shock under a Taylor rule. Panel b shows the the optimal response to a cost-push shock. The the full menu cost model is plotted in blue solid, the simplified menu cost model is plotted in dash-dotted blue, the full Calvo in dashed red.

the menu cost model, this equivalence breaks down at inflation levels such as those seen in 2022, when inflation reached approximately 10 percent.

The nonlinearity of the sacrifice ratio is the main reason behind the "strike while the iron is hot" result also in the full model. To show this, we have rerun our optimal Ramsey policy exercise in the full model combining the menu cost framework with a counterfactual quadratic objective in the (i) inflation gap, the deviation of inflation from its optimal steady state value, and (ii) output gap with relative weights derived from the second-order approximation of the Calvo model (Woodford 2003). The objective approximates the true objective in the nonlinear Calvo framework well. The results are shown in Figure 2 (yellow dotted line). The figure shows that, in line with the results of the analogous exercise in the simple model, the inflation response is similar, even more nonlinear under this counterfactual scenario than the baseline. This confirms that the key reason behind the nonlinearity of the target relationship is the nonlinearity of the Phillips relationship also in the full model.

### 4.4. Robustness and sensitivity analysis

We now show the robustness of the non-linear "strike while the iron is hot" optimal monetary policy. In particular, we explore its robustness in an extension of the Golosov and Lucas (2007) model, the CalvoPlus model (Nakamura and Steinsson 2008), and its robustness to alternative parameter choices.

*CalvoPlus model.* CalvoPlus model is a variation of the canonical menu cost model, where the menu cost is stochastic: price adjustment is free with probability $\alpha$ (as in the Calvo (1983) model) and takes a positive value ($\eta$) with probability $1 - \alpha$. This setup introduces small price changes, and therefore, improves the model's ability to fit better the distribution of price changes. At the macro level, by reducing the selection of large price changes, it increases the real effects of monetary policy and brings it closer to time-series evidence Nakamura and Steinsson 2010. In the context of our analysis, the framework affects the nonlinearity of the Phillips relationship, which raises concerns that it may potentially modify the monetary policy prescriptions already discussed. However, we show that this is not the case. This result supports the robustness of the strike-while-the-iron-is-hot policy conclusion in a realistic extension of the canonical Golosov-Lucas model.

(a) Target relationship (b) Phillips relationship

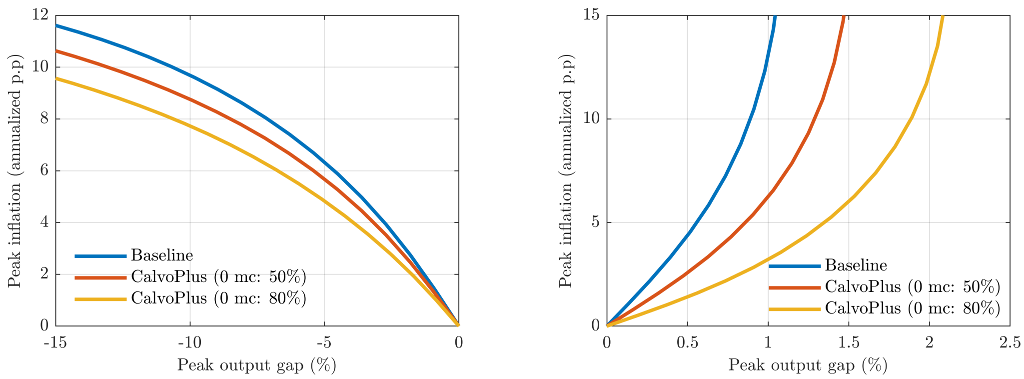


FIGURE 7. Target and Phillips relationships in the baseline and in the CalvoPlus model

The figures contrast the relationship between peak inflation and peak output gap in the baseline model versus the CalvoPlus model under optimal policy and various sizes of cost-push shocks (target relationship, panel a) and under a Taylor rule and various sizes of monetary policy easing shocks (Phillips relationship, panel b). Two parameterizations of the CalvoPlus model are presented: (i) the probability of a zero menu cost is 50%; and (ii) 80%.

Appendix G presents the CalvoPlus setup. Two parametrizations are presented: (i) the probability of zero menu cost is 50%; and (ii) 80%. We recalibrate the menu cost $\eta$ and the dispersion of idiosyncratic shocks $\sigma$ parameters to match in the steady state the average frequency in the U.S. data, and match a 20% frequency increase at a 10% inflation rate.

Results are presented in Figure 7. Panel (a) of the figure shows the target relationship between the peak inflation rate and the peak output gap under optimal policy. The figure shows that the relationship is robust in the CalvoPlus model: strike-while-the-iron-is-hot policy is still optimal. Notably, the extent of nonlinearity *increases* with the calvo parameter ($\alpha$): the higher the probability of free price changes, the more anti-inflationary optimal policy should be. This happens, even though, as we have seen above, the target relationship is near linear in the Calvo model. This result is less surprising if we notice that the Calvo parameter in the CalvoPlus model brings the baseline menu cost model closer to the Calvo model for small shocks, but influences its response to large shocks much less.

Panel (b) shows the relationship between peak inflation and peak output gap under a Taylor rule and a monetary policy shock in the three models - the Phillips relationship discussed above. The figure shows that the CalvoPlus model can substantially reduce the slope of the Phillips relationship, bringing the model closer to the Calvo (1983) model after small shocks, also improving the realism of the framework (Nakamura and Steinsson 2010). At the same time, the CalvoPlus framework still implies a highly nonlinear Phillips relationship, with a slope that increases *even faster* than the baseline for similar increases in the inflation rate. This happens because, as the shocks become larger and the frequency of adjusters increases, the share of free adjusters among all the adjusters mechanically decreases, bringing the model closer to the canonical Golosov and Lucas (2007) framework. The increase in the relative frequency of free versus costly adjustments leads to an additional source of nonlinearity in the Phillips relationship. This higher nonlinearity further reduces the sacrifice ratio of disinflation in CalvoPlus models with higher Calvo parameters, so making a stricter anti-inflationary stance optimal.

*Alternative parameterizations.* Panel (a) of Figure 8 contrasts the target relationship between annualized peak inflation and the peak output gap under Ramsey optimal policy in the baseline with alternatives with varying degrees of persistence of the cost-push shock ($\rho_\tau = 0.75, 0.99$); and with alternative values of the elasticity of substitution parameter ($\epsilon = 3, 11$). It also shows straight lines with slope $-1/\epsilon$ for $\epsilon = 3, 7, 11$. The figure shows that (i) the target relationship is influenced by the persistence of the underlying shock, but the variation is quantitatively small. Furthermore, (ii) the elasticity of substitution plays a key role in determining the slope of the target relationship. For small shocks, the slope of the target relationship is quantitatively close, but not identical to the $-1/\varepsilon$, which is the slope of the target rule, the relationship between inflation and the change in the output gap, in the linearized Calvo model. Lastly, (iii) the qualitative features of the nonlinearity after large shocks are robust: it is optimal to strike while the iron is hot for a wide range of parameter values.

Panel (b) of Figure 8 shows the robustness of the Phillips relationship. The figure reports the relationship between the peak effect of annualized inflation and the peak output gap for different i.i.d. monetary policy shocks of varying sizes as panel (b) of Figure 6. It reports how the relationship changes when varying the elasticity of substitution parameter $\epsilon = 3, 11$[34]. The figure shows that the relationship is robust and stays nonlinear across the relevant parameter space.

## 5. Optimal monetary policy: additional results

We now proceed to investigate additional results: optimal long-run inflation, optimal monetary policy to an aggregate productivity shock, and time-inconsistency of the Ramsey optimal monetary policy (commitment).

[34]We recalibrate the menu cost and the idiosyncratic quality shock volatility such that the steady state frequency stays constant across calibrations and it generates 20% frequency at 10% inflation.

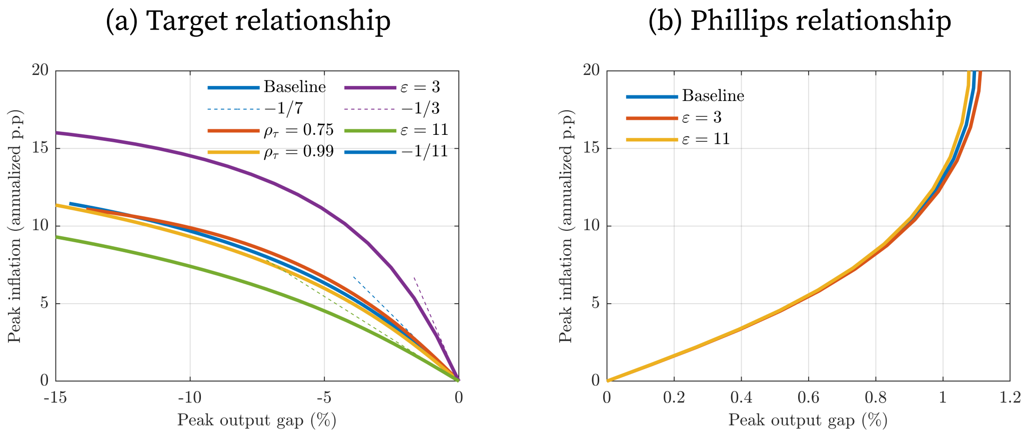


FIGURE 8. Robustness of the target and Phillips relationships for alternative parameters

The figures recreate the relationship between peak inflation and peak output gap under optimal policy and cost-push shocks (target relationship, panel a) and Taylor rule under monetary policy shock (Phillips relationship, panel b) for alternative parameter values. They show sensitivity to various elasticity of substitution parameters ($\epsilon = 3, 11$) and various persistence values of the cost push shock ($\rho_\tau = 0.75, 0.99$).

### 5.1. The steady state under the optimal policy

The solution of the Ramsey planner's problem has a steady state featuring a slightly positive inflation of 0.07%.[35] This is different from the standard New Keynesian model with Calvo pricing (Galí 2008), where the optimal inflation in the Ramsey steady state is zero. The value of inflation in the Ramsey steady state in the menu cost model is very close to the value of steady-state inflation that maximizes steady-state welfare, which in turn is also very close to the value of inflation that minimizes the frequency of price adjustments.

What explains the positive optimal inflation? The key is the asymmetry of the profit function (2.2). For a firm, a negative price gap is more undesirable than a positive price gap of the same size because a negative price gap $-x$ leads to much larger sales at a markup loss of $-x$, while the positive price gap $x$ leads to only somewhat smaller sales at a markup gain of $x$. This implies that the $(S, s)$ band is asymmetric: the lower threshold $s_t$ is closer to the optimal price than the upper one $S_t$ (see Figure 9). Thus, in the zero inflation steady state, there is more mass of firms close to the lower threshold of the inaction band than to the upper threshold. As a result, there are more upward than downward price adjustments. Small positive inflation raises the optimal reset price $p^*$ and shifts the $(S, s)$ band leftwards and thus reduces the number of upward price movements by more than it increases the number of downward price movements. The frequency of price adjustments decreases and, with it, the distortions caused by menu costs. Quantitatively, this effect is small but not negligible.

[35] In our numerical exploration, we have only found a single steady state.

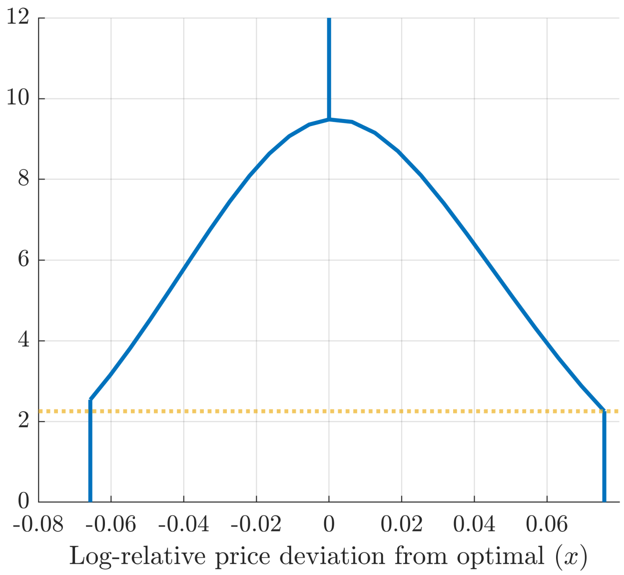


FIGURE 9. Steady-state price-gap density.

The figure displays the steady-state price-gap density $g(x)$ with zero inflation. The dashed yellow line indicates the mass of firms at the upper threshold of the $(S, s)$ band.

### 5.2. Timeless optimal monetary response to TFP shocks

Next, we consider TFP shocks, which affect the efficient allocation. In the standard New Keynesian model with Calvo prices, the response to such shocks is characterized by strict price stability: the central bank steers real interest rates to replicate the path of natural interest rates, which leads to inflation and the output gap remaining at zero. This is commonly known as the "*divine coincidence*" (Blanchard and Galí 2007).

A version of the divine coincidence also holds in our economy. As we have shown in Section 5.1, the Ramsey plan features a positive level of trend inflation in the long run. In response to a TFP shock, optimal timeless commitment policy keeps inflation at its steady-state level. We prove this formally in Appendix D. As inflation remains constant, the frequency of repricing also stays constant. In other words, the optimal policy offsets the dynamic impact of the efficient shocks in a form of "dynamic divine coincidence".

The conclusion is that strict targeting of the optimal steady-state inflation rate simultaneously minimizes inefficient output fluctuations and the costs of nominal rigidities. Notice that the shape of the Phillips curve plays no role in this result and thus the prescription is the same for small and large shocks.

### 5.3. Time-0 problem

We now turn to investigating the time inconsistency of optimal policy. To assess its magnitude, we solve the optimal policy problem, starting from the price distribution in the Ramsey steady state, assuming that the central bank faces no previous pre-

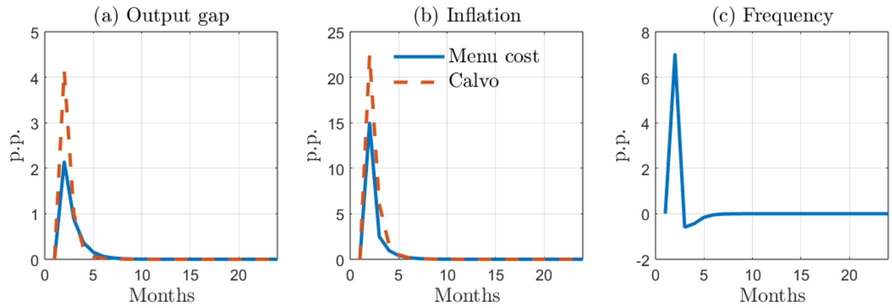


FIGURE 10. Time inconsistency of the optimal policy.

The figure compares the time-0 optimal policies in the menu cost model and in the Calvo model.

commitment. In this case, the Lagrange multipliers associated with forward-looking equations are initially set to zero. This problem is often referred to as the "time-0 problem" (Woodford 2003).

The solid blue lines in Figure 10 show the time path under the optimal policy. The labor subsidy is set to zero in this exercise, which, therefore, ceases to offset any markup distortions caused by the firms' market power. The optimal policy is time-inconsistent: without pre-commitment, the central bank engineers a temporary expansion. Thereby, it raises welfare by bringing output closer to its efficient level at a cost of elevated pricing distortions arising from the higher inflation.

The dashed red line on Figure 10 shows the equivalent time-0 response in the Calvo model. The figure shows that the incentive to surprise is substantially weaker in the menu cost model: both the inflation and output gap increases are smaller relative to the Calvo model. The reason is that the price level becomes more flexible in the state dependent model: the unexpected easing causes a sizable inflation spike, which causes an increase in the frequency of price changes. As a result, the output gap increases by less than it would under exogenous frequency. That is, the output boost from a given amount of inflation is lower than under Calvo. Since, as we saw before, the planners objective function isn't significantly different than under Calvo, the social planner thus eases less aggressively.

There is a countervailing force that raises the time inconsistency in our baseline model relative to the Calvo model. Namely, the markup distortions are higher, as discussed below, and a labor subsidy of $\tau = 1/\epsilon$ is insufficient to bring the distortions caused by the average markup to zero, as it is the case in the Calvo model. A time-0 optimal policy, therefore, stays time inconsistent even with a $\tau = 1/\epsilon$ labor subsidy (not shown). The optimal policy easing in this scenario, however, is two orders of magnitude smaller than those under no labor subsidy. Therefore, this channel is too weak to counteract the opposite effect caused by the more flexible price level detailed above.

A corollary to the negligibility of the time inconsistency with an appropriate labor subsidy is that the analysis in the previous sections, where we adopted a timeless perspective, would go through without any quantitatively relevant changes also if we adopted a time-0 perspective.

## 6. Conclusion

This paper characterizes the Ramsey optimal monetary policy in a canonical menu cost model. We find that in the presence of large cost-push shocks, optimal monetary policy should commit to quelling inflation more aggressively than what the standard New Keynesian model prescribes. Along the trajectory of optimal commitment, the central bank utilizes the reduction in the sacrifice ratio, leading to lower inflation at the cost of a slightly larger output decline. Optimal policy thus *strikes while the iron is hot.* This policy prescription diverges markedly from that of the standard New Keynesian model with exogenous timing of price adjustment, which fails to capture such nonlinear dynamics. When confronted with TFP shocks, our findings indicate that the optimal policy in the menu cost model involves a commitment to full price stability, akin to the standard New Keynesian model.

In sum, our research underscores the importance of an aggressive anti-inflationary policy by the central bank in the face of large shocks. By committing to policies that curb inflation and stabilize the repricing frequency, the central bank can deliver a more favorable macroeconomic outcome. Our analysis is confined to the case of nominal price rigidities in the seminal fixed menu cost price-setting model; we leave for future research the interaction with wage rigidities and assessment of optimal policy in more complex and realistic price-setting frameworks.

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# APPENDIX

## Appendix A. Optimality condition of the reset price

If the value function is convex, the optimal reset price is fully characterized by the system of first-order conditions in Section 2.2.[36] This appendix presents the derivation of $V_t'(0)$. To start, we reproduce here the value function presented in equation (16):

$$\begin{aligned} V_t(x) &= \Pi(x, p_t^*, w_t, A_t) \\ &+\mathbb{E}_t\left[\left(1-\lambda_{t+1}\left(x-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}^*\right)\right)\Lambda_{t,t+1}V_{t+1}(x-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}^*)\right] \\ &+\mathbb{E}_t\left[\lambda_{t+1}\left(x-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}^*\right)\Lambda_{t,t+1}\left(V_{t+1}(0)-\eta w_{t+1}\right)\right]. \end{aligned}$$

Since the only source of uncertainty is the idiosyncratic shocks, $V_t(x)$ becomes

$$\begin{aligned} V_t(x) &= \Pi(x, p_t^*, w_t, A_t) \\ &+\Lambda_{t,t+1}\int\left[\left(1-\lambda_{t+1}\left(x-\sigma_{t+1}\varepsilon-\pi_{t+1}^*\right)\right)V_{t+1}(x-\sigma_{t+1}\varepsilon-\pi_{t+1}^*)\phi(\varepsilon)\right]d\varepsilon \\ &+\Lambda_{t,t+1}\left(V_{t+1}(0)-\eta w_{t+1}\right)\int\left[\lambda_{t+1}\left(x-\sigma_{t+1}\varepsilon-\pi_{t+1}^*\right)\phi(\varepsilon)\right]d\varepsilon, \end{aligned}$$

where $\phi(\cdot)$ denotes the standard normal p.d.f. The term $x-\sigma_{t+1}\varepsilon-\pi_{t+1}^*$ is the state of the firm at $t+1$, conditional on the state $x$ at $t$ and the realization $\varepsilon_{t+1}=\varepsilon$ of the shock at $t+1$. Denoting the state at $t+1$ as $x' \equiv x-\sigma_{t+1}\varepsilon-\pi_{t+1}^*$, such that $\varepsilon \equiv \frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}$, and applying the corresponding change of variable to the integral yields

$$\begin{aligned} V_t(x) &= \Pi(x, p_t^*, w_t, A_t)+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\int\left[\left(1-\lambda_{t+1}(x')\right)V_{t+1}(x')\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx' \\ &+\left(V_{t+1}(0)-\eta w_{t+1}\right)\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\int\left[\lambda_{t+1}(x')\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx'. \end{aligned}$$

The probability of updating a price $\lambda_{t+1}$ given any state of nature is either 0 in the "inaction region" or 1 otherwise. Defining the inaction region as the $(s_t, S_t)$ band, we restrict the first integral in the latter expression. We also replace the second integral, which is the probability mass of updating the price, by 1 minus the probability mass of not updating the price. Thus:

$$\begin{aligned} V_t(x) &= \Pi(x, p_t^*, w_t, A_t)+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[V_{t+1}(x')\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx' \\ &+\Lambda_{t,t+1}\left(V_{t+1}(0)-\eta w_{t+1}\right)\left\{1-\frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)dx'\right\} \\ &= \Pi_t(x)+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[V_{t+1}(x')\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx' \\ &+\Lambda_{t,t+1}\left(V_{t+1}(0)-\eta w_{t+1}\right)\left\{1-\left[\Phi\left(\frac{x-s_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)-\Phi\left(\frac{x-S_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]\right\} \end{aligned}$$

[36] We verify convexity ex post.

where $\Phi(\cdot)$ denotes the standard normal c.d.f. and, to simplify notation, we suppress the dependence of $\Pi$ on aggregate variables.

Finally, taking the derivative of $V_t(x)$ with respect to $x$ and reformulating, we get $V_t'(x)$:

$$\begin{aligned}
V_t'(x) &= \Pi_t'(x) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \frac{\partial \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right) dx'}{\partial x} \\
&\quad + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left(\phi\left(\frac{x - S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{x - s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right)\right)(V_{t+1}(0) - \kappa w_{t+1}) \\
&= \Pi_t'(x) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \frac{\partial \phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x} dx' \\
&\quad + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left(\phi\left(\frac{x - S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{x - s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right)\right)(V_{t+1}(0) - \kappa w_{t+1})
\end{aligned}$$

which must be evaluated at $x = 0$.

## Appendix B. Comparison of static models and full model

This table compares the static Calvo model, the static Calvo model with idiosyncratic shocks to firm quality, the static Golosov-Lucas model and the dynamic Golosov Lucas model equation by equation. Note that the support of the distribution and value functions in the dynamic model is $x$, while it is $p$ in the static model, where $p = p^* + x$ .

| Expression | Calvo | Calvo with iid Shocks | Static Golosov-Lucas | Dynamic Golosov-Lucas |
|---|---|---|---|---|
| Household Labor Supply | | | $C = w$ | |
| Price Level | $1 = (1-\theta)\left(\frac{1}{e^{\pi}}\right)^{1-\epsilon} + \theta\, p^*$ | $1 = (1-\theta)\int_{-\infty}^{\infty} e^{p(1-\epsilon)} g^c(x;\pi)\, dp + \theta\, e^{p^*(1-\epsilon)}$ | $1 = \int_s^S e^{p(1-\epsilon)} g^c(p;\pi)\, dp + g^0\, e^{p^*(1-\epsilon)}$ | $1 = \int_s^S e^{(x+p_t^*)(1-\varepsilon)} g_t(x)\, dx$ |
| Price Star | | $p^* = \frac{\epsilon}{\epsilon-1}(1-\tau)w$ | | $0 = \Pi_t'(0) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \frac{\partial\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x}\, dx'$ $+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right)\left(V_{t+1}(0) - \eta\, w_{t+1}\right)$ |
| Labor Clearing Condition | $N = C\left[\theta\,(p^*)^{-\varepsilon} + (1-\theta)\left(\frac{1}{e^{\pi}}\right)^{-\varepsilon}\right]$ | $N = C\left(\int_s^S e^{p(-\epsilon)} g^c(p;\pi)\, dp + g^0\, e^{p^*(-\epsilon)}\right)$ | $N = C\left(\int_{-\infty}^{+\infty} e^{p(-\epsilon)} g^c(p;\pi)\, dp + g^0\, e^{p^*(-\epsilon)}\right) + \eta\, g^0$ | $N = \frac{C_t}{A_t}\left(\int_{s_t}^{S_t} e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p)\, dx + g_t^0\, e^{p_t^*(-\epsilon)}\right) - \upsilon\,\eta\, g_t^0$ |
| Lower Band Condition | - | - | $(p^*)^{1-\varepsilon} - (1-\tau)w\,(p^*)^{-\varepsilon} C - \eta\, C$ $= s^{1-\varepsilon} - (1-\tau)w\, s^{-\varepsilon} C$ | $V_t(0) - \eta\, w_t = V_t(s_t)$ |
| Upper Band Condition | - | - | $(p^*)^{1-\varepsilon} - (1-\tau)w\,(p^*)^{-\varepsilon} C - \eta\, C$ $= S^{1-\varepsilon} - (1-\tau)w\, s^{-\varepsilon} C$ | $V_t(0) - \eta\, w_t = V_t(S_t)$ |
| Price Frequencies | $g_0 = \theta$ | $g_0 = \theta$ | $g_0 = 1 - \int_s^S g^c(p+\pi)\, dp$ | $g_t^0 = 1 - \int_{s_t}^{S_t} g_t^c(x)\, dx$ |
| Price-Gap Density | - | - | - | $g_t^c(x) = \frac{1}{\sigma_t}\int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1})\phi\left(\frac{x_{-1} - x - \pi_t^*}{\sigma_t}\right) dx_{-1} + g_{t-1}^0\, \phi\left(\frac{-x-\pi_t^*}{\sigma_t}\right)$ |
| Bellman Equation | - | - | - | $V_t(x) = \Pi(x, p_t^*, w_t, A_t)$ $+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x')\phi\left(\frac{(x-x') - \pi_{t+1}^*}{\sigma_{t+1}}\right) dx'$ $+ \Lambda_{t,t+1}\left(1 - \frac{1}{\sigma_{t+1}} \int_{s_{t+1}}^{S_{t+1}} \phi\left(\frac{(x-x') - \pi_{t+1}^*}{\sigma_{t+1}}\right) dx'\right)$ $\times\left(V_{t+1}(0) - \eta\, w_{t+1}\right)$ |
| Utility/Welfare | | | $u = \log(C) - N$ | |
| Variable Count | 6 | 8 | 8 | 10 |
| Equation Count | 5 | 5 | 7 | 9 |

TABLE A1. Comparison of Different Models

## Appendix C. Efficiency and welfare analysis

### C.1. Efficient and natural level of output

This appendix derives efficient output, efficient real interest rate, and natural output.

**Efficient output**. We obtain it as the solution to a social planning problem. The problem maximizes household welfare in equation (1) subject to (i) the aggregate consumption equation (3), (ii) aggregate labor supply in ($N_t = \int_i N_t(j)$) and (iii) product-level production functions in (10) with respect to product-level consumption and labor ($C_t(j), N_t(j), j \in [0,1], t = 0,1,2,\ldots$).

After some algebra, the optimization problem simplifies to

$$\max_{N_t(j)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[A_t\left(\int N_t(j)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}\right]^{1-\gamma}}{1-\gamma} - \upsilon \int N_t(j) di,$$

subject to $\int N_t(j)di = N_t$. The solution implies that the efficient output fluctuates with aggregate productivity but is independent of demand shocks as well as of cost-push shocks. In particular, the efficient level of output is

$$Y_t^e = C_t^e = A_t N_t^e = \upsilon^{-1/\gamma} A_t^{1/\gamma}. \tag{A1}$$

For our parametrization, $\upsilon = 1$ and $\gamma = 1$, we thus have that

$$\begin{aligned} N_t^e &= 1, \\ C_t^e &= A_t. \end{aligned} \tag{A2}$$

The efficient labor supply is equal across products and the efficient product-level consumption varies across products $j$ inversely proportional to the product-level quality, in particular

$$\begin{aligned} N_t^e(j) &= N_t^e \\ C_t^e(j) &= \frac{A_t N_t^e}{A_t(j)}. \end{aligned}$$

**Efficient real interest rate**. It is implicitly defined by the Euler equation after substituting in efficient consumption:

$$r_t^e = -\log\beta - \gamma(1-\rho_A)\log A_t$$

**Natural output**. It is defined as the counterfactual output with flexible prices. Under flexible prices, firms maximize their real profit function (2.2) in each period $t$ by choosing

$$\frac{P_t^n(j)}{A_t(j)P_t^n} = \frac{\epsilon}{\epsilon-1}(1-\tau_t)\frac{w_t^n}{A_t}.$$

The expression implies that the quality-adjusted relative price is homogeneous across products $j$. Together with the definition of the price-level in equation (6) in the main text, this implies that the natural level of the quality-adjusted log relative price is zero ($p_t(j) = 0$). Or equivalently, the natural level of relative price is equal to the quality: $P_t^n(j)/P_t^n = A_t(j)$. The product-level demand function and the unit quality-adjusted relative price implies that product-level natural consumption is inversely proportional to the quality of product $j$:

$$C_t^n(j) = \frac{1}{A_t(j)} C_t^n.$$

Furthermore, the natural real wage, output and labor are given by the following closed-form expressions:

$$\begin{aligned} w_t^n &= A_t \frac{\epsilon - 1}{\epsilon} \frac{1}{1 - \tau_t}, \\ Y_t^n &= C_t^n = \left(\frac{w_t^n}{\upsilon}\right)^{1/\gamma}, \\ N_t^n &= \frac{Y_t^n}{A_t}. \end{aligned}$$

Notably, the productivity shock affects the natural and the efficient output similarly, but the cost-push (labor-tax) shocks only affect the natural level of output.

### C.2. Welfare decomposition

This appendix derives the welfare decomposition presented in equation (28) in the main text. We start by obtaining expressions that are used at the end for the welfare decomposition.

**Markups**. The real marginal cost of firm $j$ is

$$MC_t(j) = \frac{\partial\left((1 - \tau_t) w_t N_t(j)\right)}{\partial Y_t(j)} = \frac{(1 - \tau_t) w_t A_t(j)}{A_t},$$

where we have used that $N_t(j) = A_t(j) Y_t(j)/A_t$.

The (log-) markup $\mu_t(j)$ is the (log-) difference between the relative price and the real marginal cost:

(A3) $$\mu_t(j) = \log \frac{P_t(j)}{P_t} - \log \frac{(1 - \tau_t) w_t A_t(j)}{A_t} = \log \frac{P_t(j)}{A_t(j) P_t} - \log \frac{(1 - \tau_t) w_t}{A_t} = p_t(j) - mc_t,$$

where $p_t(j)$ is the quality-adjusted relative price and

$$mc_t \equiv \log\left(MC_t(j)/A_t(j)\right) = \log\left((1 - \tau_t) w_t / A_t\right) = \log(1 - \tau_t) - \log A_t + \log \upsilon + \gamma \log Y_t,$$

is the 'aggregate component' of the marginal cost. Notice that we have employed eq. (7).

**Aggregate markup and output**. The aggregate (log-) markup $\overline{\mu}_t$ is

$$\overline{\mu}_t = \log\left(\int e^{\mu_t(j)(1-\epsilon)}dj\right)^{\frac{1}{1-\epsilon}} = \log\left(\int \frac{e^{p_t(j)(1-\epsilon)}}{e^{mc_t(1-\epsilon)}}dj\right)^{\frac{1}{1-\epsilon}} = \log\frac{1}{e^{mc_t}}\left(\int e^{p_t(j)(1-\epsilon)}dj\right)^{\frac{1}{1-\epsilon}} = -mc_t,$$

where we used the observation that the average quality-adjusted relative price is one (eq. 6). Therefore

$$\overline{\mu}_t = -\log(1-\tau_t) + \log A_t - \log\upsilon - \gamma\log Y_t \tag{A4}$$

or equivalently

$$e^{\overline{\mu}_t} = \frac{A_t}{\upsilon(1-\tau_t)Y_t^{\gamma}} \tag{A5}$$

expressing the tight relationship between average markup and the output.

Taking into account eq. (A1), the efficient output gap can be expressed

$$\log Y_t - \log Y_t^e = \frac{1}{\gamma}\left(-\log(1-\tau_t) - \overline{\mu}_t\right), \tag{A6}$$

which is proportional to the negative average markup.

**Markup dispersion**. The dispersion of the quality-adjusted relative prices ($\zeta_t^p$) is

$$\zeta_t^p = \int e^{p_t(j)(-\epsilon)}g(p_t(j))dj = \int e^{(\mu_t(j)+mc_t)(-\epsilon)}g(\mu_t(j)+mc_t)dj = \int e^{(\mu_t(j)-\overline{\mu}_t)(-\epsilon)}g(\mu_t(j)-\overline{\mu}_t)dj \equiv \zeta_t^{\mu-\overline{\mu}}$$

where $\zeta_t^{\mu-\overline{\mu}}$ is the dispersion of the demeaned markups which equals price dispersion.

**Welfare**. Finally, consider the case with $\gamma = 1$. We can express the difference between welfare ($W_t$) from the welfare in the efficient equilibrium ($W_t^e$) subject to ashock in period 0 as

$$\begin{aligned}
W_0 - W_0^e = \sum_{t=0}^{\infty}\beta^t\left(U_t - U_t^e\right) &= \sum_{t=0}^{\infty}\beta^t\left((\log C_t - N_t) - (\log C_t^e - N_t^e)\right)\\
&= \sum_{t=0}^{\infty}\beta^t\left((\log Y_t - N_t) - (\log Y_t^e - 1)\right)\\
&= \sum_{t=0}^{\infty}\beta^t\left(-\log(1-\tau_t) - \overline{\mu}_t - \frac{C_t}{A_t}\int e^{p(-\epsilon)}g_t(p)\,dp - \eta g_t^0 + 1\right)\\
&= \sum_{t=0}^{\infty}\beta^t\left(-\log(1-\tau_t) - \overline{\mu}_t - \left(\frac{1}{e^{\overline{\mu}_t}(1-\tau_t)}\zeta_t^{\mu-\overline{\mu}} - 1\right) - \eta g_t^0\right)\\
&= \sum_{t=0}^{\infty}\beta^t\left(-\log(1-\tau_t) - \overline{\mu}_t - \left(\frac{1}{e^{\overline{\mu}_t}(1-\tau_t)} - 1\right) - \left(\frac{1}{e^{\overline{\mu}_t}(1-\tau_t)}(\zeta_t^{\mu-\overline{\mu}} - 1)\right) - \eta g_t^0\right)
\end{aligned}$$

where $U_t^e$ is the utility in the efficient equilibrium and where we have used (A2) and

$C_t = Y_t$ in line 1 and (A6) and (20) in line 2 and (A5) and $C_t = Y_t$ and $\gamma = 1$ in line 3. The final expression decomposes welfare into terms related to average markup, markup dispersion, and adjustment costs.

# Appendix D. Response to TFP shocks

This appendix proves that, in response to a TFP shock, optimal timeless commitment policy keeps inflation at its steady-state level $\pi_t = \pi$.

The planner's problem is:

$$\max_{\substack{\{g_t^c(\cdot), g_t^0, V_t(\cdot), C_t, \\ w_t, p_t^*, s_t, S_t, \pi_t^*\}_{t=0}^{\infty}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \upsilon N_t\right)$$

subject to

$$\begin{aligned}
w_t &= \upsilon C_t, \\
N_t &= \frac{C_t}{A_t}\left(\int_{s_t}^{S_t} e^{\left(x+p_t^*\right)(-\epsilon_t)} g_t^c\left(p\right)dx + g_t^0 e^{\left(p_t^*\right)(-\epsilon)}\right) - \upsilon\eta g_t^0 \\
V_t(x) &= \Pi(x, p_t^*, w_t, A_t) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[V_{t+1}(x')\phi\left(\frac{\left(x-x'\right)-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx' + \\
&\quad \Lambda_{t,t+1}\left(1 - \frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[\phi\left(\frac{\left(x-x'\right)-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx'\right)\left[\left(V_{t+1}\left(0\right) - \eta w_{t+1}\right)\right], \\
V_t\left(s_t\right) &= V_t\left(0\right) - \eta w_t, \\
V_t\left(S_t\right) &= V_t\left(0\right) - \eta w_t, \\
0 &= \Pi_t'(0) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \left.\frac{\partial\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x}\right|_{x=0} dx' \\
&\quad + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right)\left(V_{t+1}(0) - \eta w_{t+1}\right). \\
g_t^c(x) &= \frac{1}{\sigma_t}\int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1})\phi\left(\frac{x_{-1}-x-\pi_t^*}{\sigma_t}\right)dx_{-1} + g_{t-1}^0\phi\left(\frac{-x-\pi_t^*}{\sigma_t}\right), \\
g_t^0 &= 1 - \int_{s_t}^{S_t} g_t^c(x)dx, \\
1 &= \int_{s_t}^{S_t} e^{\left(x+p_t^*\right)(1-\epsilon)} g_t^c\left(x\right)dx + g_t^0 e^{\left(p_t^*\right)(1-\epsilon)}.
\end{aligned}$$

We now transform it in a convenient fashion. First, normalize the constraints involving $V_t(x)$ by $A_t$ and substitute for the wage $w_t = \upsilon C_t$ and the discount factor $\Lambda_{t,t+1} = \beta\frac{C_t}{C_{t+1}}$. With this, the constrains involving $V_t(x)$ become:

$$\begin{aligned}
\frac{V_t(x)}{A_t} &= \frac{C_t}{A_t}\left(\exp\left(x_t + p_t^*\right)\right)^{1-\epsilon} - \frac{C_t}{A_t}(1-\tau_t)\upsilon\frac{C_t}{A_t}\left(\exp\left(x_t + p_t^*\right)\right)^{-\epsilon} \\
&\quad + \beta\frac{A_{t+1}}{A_t}\frac{C_t}{C_{t+1}}\frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[\frac{V_{t+1}(x')}{A_{t+1}}\phi\left(\frac{\left(x-x'\right)-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx'
\end{aligned}$$

$$
\begin{aligned}
& +\frac{A_{t+1}}{A_t}\beta\frac{C_t}{C_{t+1}}\left(1-\frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx'\right)\left[\left(\frac{V_{t+1}(0)}{A_{t+1}}-\eta\upsilon\frac{C_{t+1}}{A_{t+1}}\right)\right],\\
\frac{V_t(s_t)}{A_t} &= \frac{V_t(0)}{A_t}-\eta\upsilon\frac{C_t}{A_t},\\
\frac{V_t(S_t)}{A_t} &= \frac{V_t(0)}{A_t}-\eta\upsilon\frac{C_t}{A_t},\\
0 &= (1-\epsilon)\frac{C_t}{A_t}\left(\exp\left(x_t+p_t^*\right)\right)^{1-\epsilon}+\epsilon\frac{C_t}{A_t}(1-\tau_t)\upsilon\frac{C_t}{A_t}\left(\exp\left(x_t+p_t^*\right)\right)^{-\epsilon}\\
& +\frac{1}{\sigma_{t+1}}\beta\frac{A_{t+1}}{A_t}\frac{C_t}{C_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\frac{V_{t+1}(x')}{A_{t+1}}\left.\frac{\partial\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x}\right|_{x=0}dx'\\
& +\frac{1}{\sigma_{t+1}}\beta\frac{A_{t+1}}{A_t}\frac{C_t}{C_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)-\phi\left(\frac{-s_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right)\left(\frac{V_{t+1}(0)}{A_{t+1}}-\eta\upsilon\frac{C_{t+1}}{A_{t+1}}\right).
\end{aligned}
$$

Second, define $\frac{V_t(x)}{A_t}\equiv\hat{V}_t(x)$, so that these constrains become

$$
\begin{aligned}
\hat{V}_t(x) &= \frac{C_t}{A_t}\left(\exp\left(x_t+p_t^*\right)\right)^{1-\epsilon}-\frac{C_t}{A_t}(1-\tau_t)\upsilon\frac{C_t}{A_t}\left(\exp\left(x_t+p_t^*\right)\right)^{-\epsilon}\\
& +\beta\frac{C_t}{A_t}\frac{A_{t+1}}{C_{t+1}}\frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[\hat{V}_{t+1}(x')\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx'\\
& +\beta\frac{C_t}{A_t}\frac{A_{t+1}}{C_{t+1}}\left(1-\frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx'\right)\left[\left(\hat{V}_{t+1}(0)-\eta\upsilon\frac{C_{t+1}}{A_{t+1}}\right)\right],\\
\hat{V}_t(s_t) &= \hat{V}_t(0)-\eta\upsilon\frac{C_t}{A_t},\\
\hat{V}_t(S_t) &= \hat{V}_t(0)-\eta\upsilon\frac{C_t}{A_t},\\
0=\hat{V}_t'(0) &= (1-\epsilon)\frac{C_t}{A_t}\left(\exp\left(x_t+p_t^*\right)\right)^{1-\epsilon}+\epsilon\frac{C_t}{A_t}(1-\tau_t)\upsilon\frac{C_t}{A_t}\left(\exp\left(x_t+p_t^*\right)\right)^{-\epsilon}\\
& +\frac{1}{\sigma_{t+1}}\beta\frac{C_t}{A_t}\frac{A_{t+1}}{C_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\hat{V}_{t+1}(x')\left.\frac{\partial\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x}\right|_{x=0}dx'\\
& +\frac{1}{\sigma_{t+1}}\beta\frac{C_t}{A_t}\frac{A_{t+1}}{C_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)-\phi\left(\frac{-s_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right)\left(\hat{V}(0)-\eta\upsilon\frac{C_{t+1}}{A_{t+1}}\right).
\end{aligned}
$$

Finally, define $\hat{C}_t=\frac{C_t}{A_t}$. The planner's problem becomes

$$
\max_{\substack{\{g_t^c(\cdot),g_t^0,\hat{V}_t(\cdot),\hat{C}_t,\\ w_t,p_t^*,s_t,S_t,\pi_t^*,L_t\}_{t=0}^{\infty}}}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left(\log\left(\hat{C}\right)+\log\left(A_t\right)-\upsilon L_t\right)
$$

$$
\hat{V}_t(x) = \hat{C}_t\left(\exp\left(x_t+p_t^*\right)\right)^{1-\epsilon}-\hat{C}_t(1-\tau_t)\upsilon\hat{C}_t\left(\exp\left(x_t+p_t^*\right)\right)^{-\epsilon}
$$

$$
\begin{aligned}
& +\beta\hat{C}_t\hat{C}_{t+1}^{-1}\frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[\hat{V}_{t+1}(x')\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx' \\
& +\beta\hat{C}_t\hat{C}_{t+1}^{-1}\left(1-\frac{1}{\sigma_{t+1}}\int_{s_{t+1}}^{S_{t+1}}\left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right]dx'\right)\left[\left(\hat{V}_{t+1}(0)-\eta\upsilon\hat{C}_{t+1}\right)\right], \\
\hat{V}_t(s_t) &= \hat{V}_t(0)-\eta\upsilon\hat{C}_t, \\
\hat{V}_t(S_t) &= \hat{V}_t(0)-\eta\upsilon\hat{C}_t, \\
0=\hat{V}_t'(0) &= (1-\epsilon)\hat{C}_t\left(\exp\left(x_t+p_t^*\right)\right)^{1-\epsilon}+\epsilon\hat{C}_t(1-\tau_t)\upsilon\hat{C}_t\left(\exp\left(x_t+p_t^*\right)\right)^{-\epsilon} \\
& +\frac{1}{\sigma_{t+1}}\beta\hat{C}_t\hat{C}_{t+1}^{-1}\int_{s_{t+1}}^{S_{t+1}}\hat{V}_{t+1}(x')\left.\frac{\partial\phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x}\right|_{x=0}dx' \\
& +\frac{1}{\sigma_{t+1}}\beta\hat{C}_t\hat{C}_{t+1}^{-1}\left(\phi\left(\frac{-S_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)-\phi\left(\frac{-s_{t+1}-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right)\left(\hat{V}(0)-\eta\upsilon\hat{C}_{t+1}\right). \\
N_t &= \hat{C}_t\left(\int_{s_t}^{S_t}e^{\left(x+p_t^*\right)(-\epsilon_t)}g_t^c(p)\,dx+g_t^0e^{\left(p_t^*\right)(-\epsilon)}\right) \\
g_t^c(x) &= \frac{1}{\sigma_t}\int_{s_{t-1}}^{S_{t-1}}g_{t-1}^c(x_{-1})\phi\left(\frac{x_{-1}-x-\pi_t^*}{\sigma_t}\right)dx_{-1}+g_{t-1}^0\phi\left(\frac{-x-\pi_t^*}{\sigma_t}\right), \\
g_t^0 &= 1-\int_{s_t}^{S_t}g_t^c(x)dx, \\
1 &= \int_{s_t}^{S_t}e^{\left(x+p_t^*\right)(1-\epsilon)}g_t^c(x)\,dx+g_t^0e^{\left(p_t^*\right)(1-\epsilon)}.
\end{aligned}
$$

Notice that TFP $A_t$ only appears in the objective in a separable way. Therefore, the redefined Ramsey policy is independent of TFP shocks. Going back to the original variables definition, this implies that under optimal policy $C_t \propto A_t$ and $V_t(x) \propto A_t$ while all other variables remain constant at their steady-state values. Thus, inflation $\pi_t$ also remains constant at its steady-state value.

## Appendix E. Computational algorithm

This appendix explains the computational method. We use a three-step approach we use to convert the original infinite-dimensional Ramsey problem into a finite-dimensional one. First, we approximate the distribution and value functions by piece-wise linear functions over a set of nodes. Second, we use endogenous nodes, such that both boundaries of the $(s_t, S_t)$ band and the optimal reset price are "on the grid". Third, given this approximation, we evaluate integrals analytically. Step one makes the problem finite dimensional. Steps two and three ensure that the approximation is accurate, smooth and computationally efficient.We explain those steps in detail below.

Once we have converted the planner's infinite-dimensional problem into a finite-dimension problem in this way, we derive the planner's first order conditions. For this we use symbolic differentiation, and in particular Dynare's `Ramsey` command. The resulting set of first order conditions is then solved in the sequence space under perfect foresight. Here we employ a standard Newton method using Dynare's `perfect foresight solver` command.

To determine the appropriate initial and terminal conditions, and an initial guess for the transition paths, we need to find the non-stochastic steady state of the model. We determine the steady state of the private equilibrium conditional on a particular value of the policy instrument $\pi$ using a standard Newton based solution method. We then use this function and exploit the linearity of the first order conditions wrt. the Lagrange multipliers to convert the high-dimensional problem of solving for the steady state into a one-dimensional problem, which is solved with a Newton solver. This last step is performed by Dynare's `steady` command. That is, we have to manually convert the problem into a finite-dimension problem and find the steady state conditional on a policy; the rest of the procedure uses Dynare.

The rest of the appendix explains those steps that are not straightforward applications of existing methods. It is organized as follows. First we explain how to make the planner's problem finite dimensional. For this purpose, we first define some useful auxiliary functions in Section E.1. Then we transform the equilibrium conditions to apply an endogenous grid and approximate the value and distribution functions by a piece-wise linear function in Section E.2. Finally, we evaluate the integrals analytically in Section E.3. The result is a discrete set of equations that can conveniently be represented in matrix form, which we summarized in Section E.4. Second, we explain how we determine the steady state in Section E.5.

### E.1. Preliminaries

To begin with, let us normalize the variable $x_t$ as

(A7)
$$\mathrm{x_t} = \begin{cases} \frac{x_t}{s_t} & \text{if } x_t < 0 \\ \frac{x_t}{S_t} & \text{else} \end{cases}$$

Under this normalization, the optimal price is at $\mathrm{x_t} = 0$, the upper boundary of the $(S, s)$ band at $\mathrm{x_t} = 1$ and the lower boundary of the $(S, s)$ band at $\mathrm{x_t} = -1$. This will later allow us to have all critical points $(s_t, S_t, p_t^*)$ on the grid. The law of motion of $\mathrm{x_t}$ conditional on not updating can be derived from $x_t = x_{t-1} - \sigma\varepsilon_t - \pi_t^*$:

(A8)
$$\mathrm{x_t} = \begin{cases} \frac{x_t}{S_t} = \frac{x_{t-1}-\sigma_t\varepsilon_t-\pi_t^*}{S_t} = \begin{cases} \frac{x_{t-1}-\sigma_t\varepsilon_t-\pi_t^*}{S_{t-1}}\frac{S_{t-1}}{S_t} = \mathrm{x_{t-1}}\frac{S_{t-1}}{S_t} - \frac{\sigma_t\varepsilon_t+\pi_t^*}{S_t} & \text{if } \mathrm{x_t} > 0, \text{if } \mathrm{x_{t-1}} > 0 \\ \frac{x_{t-1}-\sigma_t\varepsilon_t-\pi_t^*}{s_{t-1}}\frac{s_{t-1}}{S_t} = \mathrm{x_{t-1}}\frac{s_{t-1}}{S_t} - \frac{\sigma_t\varepsilon_t+\pi_t^*}{S_t} & \text{if } \mathrm{x_t} > 0, \text{if } \mathrm{x_{t-1}} < 0 \end{cases} \\ \frac{x_t}{s_t} = \frac{x_{t-1}-\sigma_t\varepsilon_t-\pi_t^*}{s_t} = \begin{cases} \frac{x_{t-1}-\sigma_t\varepsilon_t-\pi_t^*}{S_{t-1}}\frac{S_{t-1}}{s_t} = \mathrm{x_{t-1}}\frac{S_{t-1}}{s_t} - \frac{\sigma_t\varepsilon_t+\pi_t^*}{s_t} & \text{if } \mathrm{x_t} < 0, \text{if } \mathrm{x_{t-1}} > 0 \\ \frac{x_{t-1}-\sigma_t\varepsilon_t-\pi_t^*}{s_{t-1}}\frac{s_{t-1}}{s_t} = \mathrm{x_{t-1}}\frac{s_{t-1}}{s_t} - \frac{\sigma_t\varepsilon_t+\pi_t^*}{s_t} & \text{if } \mathrm{x_t} < 0, \text{if } \mathrm{x_{t-1}} < 0 \end{cases} \end{cases}$$

We now define functions to be used in the next sections to redefine the value and distribution functions. For compactness, let us adopt the notation where $\hat{s}_t(\mathrm{x_t})$ picks the respective extremes $(S, s)$ depending on the value of $\mathrm{x_t}$ following (A7). For brevity, at times we will drop the dependence on $\mathrm{x_t}$ and just write $\hat{s}_t$.

Solving (A8) for $\mathrm{x_t}$, $\mathrm{x_{t-1}}$ and $\varepsilon$ respectively, we obtain the following relations:

(A9)
$$\mathrm{x_t} = \mathrm{x_{t-1}}\frac{\hat{s}_{t-1}}{\hat{s}_t} - \frac{\sigma_t\varepsilon_t + \pi_t^*}{\hat{s}_t}$$

(A10)
$$\mathrm{x_{t-1}} = \mathrm{x_t}\frac{\hat{s}_t}{\hat{s}_{t-1}} + \frac{\sigma_t\varepsilon_t + \pi_t^*}{\hat{s}_{t-1}}$$

(A11)
$$\varepsilon_t = \frac{\hat{s}_{t-1}\mathrm{x_{t-1}} - \hat{s}_t\mathrm{x_t} - \pi_t^*}{\sigma_t} \equiv h(\mathrm{x_{t-1}}, \mathrm{x_t})$$

where we have defined $h(\mathrm{x_{t-1}}, \mathrm{x_t})$ for later use.

### E.2. Approximating the distribution and value functions by piecewise linear functions on an endogenous grid

Now we redefine the value and distribution functions over the variable $\mathrm{x}$ and approximate them by piece-wise linear functions. The original infinite dimensional problem of the planner are laid out in Section 3.1. In the following, we consider each of the equations containing the distribution and value functions one by one.

### E.2.1. Distribution

The distribution function is given by

$$g_t(x) = (1-\lambda_t(x)) \int g_{t-1}(x+\sigma_t\varepsilon_t+\pi_t^*)d\xi(\varepsilon)+\delta(x)\int \lambda_t(\tilde{x})\left(\int g_{t-1}(\tilde{x}+\sigma_t\varepsilon+\pi_t^*)d\xi(\varepsilon)\right)d\tilde{x}$$

with

$$\int_{s_t}^{S_t} g_t(x)dx = 1 \tag{A12}$$

where $\delta(x)$ is the Dirac Delta function that captures the mass point of those firms who update their prices.

We split the distribution into the continuous distribution of agents who do not update their prices plus a mass point of updaters at $x = 0$ (this is already reflected in Section 3.1):

$$\begin{aligned} g_t^c(x) &= (1-\lambda_t(x))\int g_{t-1}^c(x+\sigma_t\varepsilon+\pi_t^*)d\xi(\varepsilon), \\ g_t^0 &= \int \lambda_t(\tilde{x})\int g_{t-1}^c(\tilde{x}+\sigma_t\varepsilon-\pi_t^*)d\xi(\varepsilon)d\tilde{x}. \end{aligned}$$

Furthermore, we use equation (A12) to express the latter expression as:

$$g_t^0 = 1-\int_{s_t}^{S_t} g_t^c(x)dx.$$

Now rewrite it using the newly defined re-normalized $\mathrm{x}$ where $x = \mathrm{x}\hat{s}_t$ as in equation (A7): define $g_t(\mathrm{x}\hat{s}_t) \equiv \mathrm{g}_t(\mathrm{x})$ and $g_t^c(\mathrm{x}\hat{s}_t) \equiv \mathrm{g}_t^c(\mathrm{x})$ and, with a slight abuse of notation, $\lambda_t(\mathrm{x}\hat{s}_t) \equiv \lambda_t(\mathrm{x})$ and write

$$\mathrm{g}_t^c(\mathrm{x}) = (1-\lambda_t(\mathrm{x}))\int \mathrm{g}_{t-1}\left(\frac{\mathrm{x}\hat{s}_t+\sigma_t\varepsilon+\pi_t^*}{\hat{s}_{t-1}}\right)d\xi(\varepsilon), \tag{A13}$$

$$\mathrm{g}_t^0 = 1-\int_{-1}^{1}\mathrm{g}_t^c(\mathrm{x})\,\hat{s}_t(\mathrm{x})d\mathrm{x}. \tag{A14}$$

Note that for the latter expression for $\mathrm{g}_t^0$ we have applied a change of variable to the integral. In particular, we have used the following substitution:

$$\begin{aligned} \int_{s_t}^{S_t} g_t^c(x)dx &= \int_{s_t}^{S_t} g_t^c(\mathrm{x}\hat{s}_t(\mathrm{x}))d\mathrm{x}\hat{s}_t(\mathrm{x}) \\ &= \int_{s_t}^{S_t}\mathrm{g}_t^c(\mathrm{x})d\mathrm{x}\hat{s}_t(\mathrm{x}) = \int_{s_t/\hat{s}_t(\mathrm{x})}^{S_t/\hat{s}_t(\mathrm{x})}\hat{s}_t(\mathrm{x})\mathrm{g}_t^c(\mathrm{x})d\mathrm{x} = \int_{-1}^{1}\hat{s}_t(\mathrm{x})\mathrm{g}_t^c(\mathrm{x})d\mathrm{x}. \end{aligned}$$

Next we will also change the variable in the integral in the equation for $\mathrm{g}_t^c(\mathrm{x})$ (A13). This change of variable is a bit more involved, so we derive it in detail here. First, we re-express (A13) as

$$g_t^c(x) = (1 - \lambda_t(x)) \int g_{t-1}\left(\frac{x\hat{s}_t + \sigma_t \varepsilon + \pi_t^*}{\hat{s}_{t-1}}\right)\phi(\varepsilon)d\varepsilon.$$

where $\phi(\cdot)$ is the standard normal pdf.

Second, we define the value of the shock $\varepsilon$ necessary to get from a price gap of $x_{t-1} = 0$ to a price gap of $x_t$ as

$$\begin{aligned}\varepsilon_t^* &\equiv \varepsilon_t \in \mathbb{R}|0 = x_t\hat{s}(x_t) + \sigma_t\varepsilon_t + \pi_t^* \\ &= h(0, x_t)\end{aligned}$$

and then we split the integral in two parts at $\varepsilon_t^*$

$$\begin{aligned}g_t^c(x) &= (1 - \lambda_t(x)) \int^{\varepsilon_t^*} g_{t-1}\left(\frac{x\hat{s}_t + \sigma_t\varepsilon + \pi_t^*}{\hat{s}_{t-1}}\right)\phi(\varepsilon)d\varepsilon \\ &+ (1 - \lambda_t(x)) \int_{\varepsilon_t^*} g_{t-1}\left(\frac{x\hat{s}_t + \sigma_t\varepsilon + \pi_t^*}{\hat{s}_{t-1}}\right)\phi(\varepsilon)d\varepsilon,\end{aligned}$$

Since, for a realization $\varepsilon$ of the shock at $t$,

$$\hat{s}(x_{t-1})\,x_{t-1} = x_t\hat{s}(x_t) + \sigma_t\varepsilon + \pi_t^*, \tag{A15}$$

we have

$$d\varepsilon = \left(\frac{\hat{s}(x_{t-1})}{\sigma_t} + \frac{x_{t-1}}{\sigma_t}\frac{d\hat{s}(x_{t-1})}{dx_{t-1}}\right)dx_{t-1}.$$

In each of the two intervals over which the two integrals are defined, the mapping (A15) is continuous and $\frac{d\hat{s}(x_{t-1})}{dx_{t-1}} = 0$. Thus we can implement a change of variable from $\varepsilon$ to $x_{t-1}$ in both integrals:

$$\begin{aligned}g_t^c(x) &= (1 - \lambda_t(x)) \int^{\varepsilon_t^*} \frac{s_{t-1}}{\sigma_t} g_{t-1}(x_{t-1})\phi\left(\frac{s_{t-1}x_{t-1} - \hat{s}_t x - \pi_t^*}{\sigma_t}\right)dx_{t-1} \\ &+ (1 - \lambda_t(x)) \int_{\varepsilon_t^*} g_{t-1}(x_{t-1})\phi\left(\frac{S_{t-1}x_{t-1} - \hat{s}_t x - \pi_t^*}{\sigma_t}\right)\frac{S_{t-1}}{\sigma_t}dx_{t-1}.\end{aligned}$$

Finally, pasting the two integrals together again, re-denoting $x_{t-1}$ by $x'$ and using $h(x', x)$

$$g_t^c(x) = (1 - \lambda_t(x)) \int \frac{\hat{s}_{t-1}(x')}{\sigma_t} g_{t-1}(x')\phi\left(h(x', x)\right)dx'$$

This concludes the change of variables.

The end of period distribution has mass on the $(s, S)$ band, i.e. in the range $x \in$

$[-1, 1]$. We can thus restrict the boundaries of the integral accordingly:

$$g_t^c(x) = (1 - \lambda_t(x)) \int_{-1}^{1} \frac{\hat{s}_{t-1}(x')}{\sigma_t} g_{t-1}(x') \phi\left(h(x', x)\right) \, dx'.$$

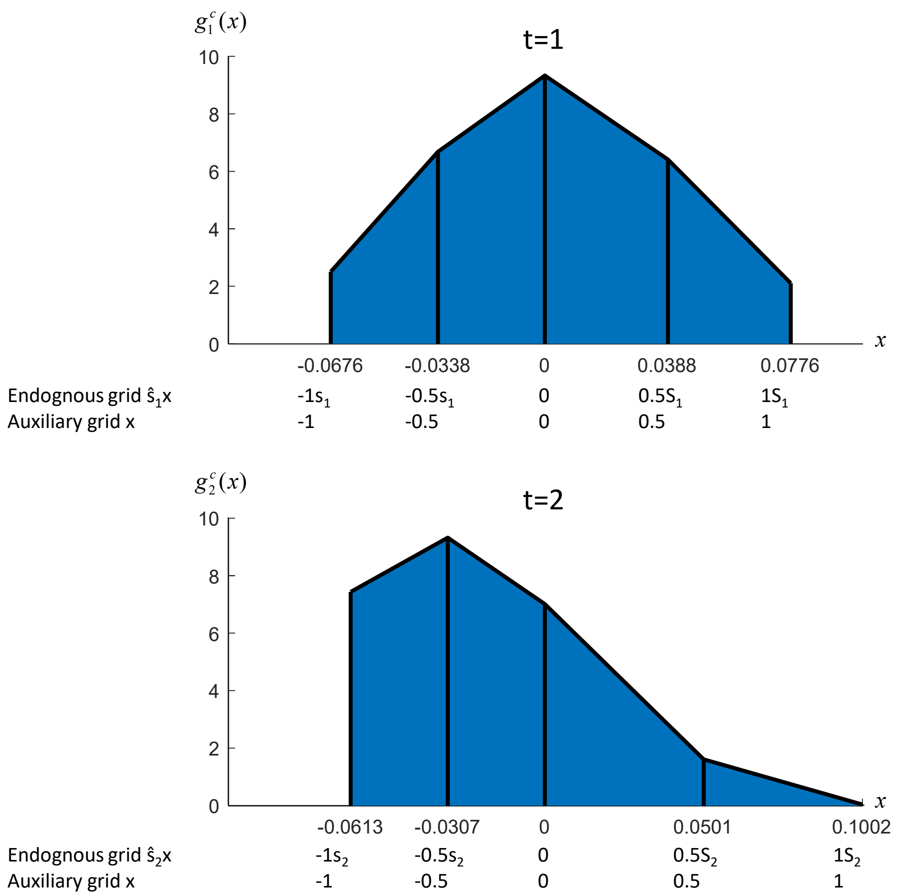


FIGURE A1. This figure schematically explains the linear interpolation with an endogenous grid. It shows the piece-wise linearly approximated distribution $g_t^c(x)$ at two points in time, $t = 1$ and $t = 2$. The thresholds of the $(S, s)$ band are not symmetric around 0 and differ across time. The endogenous grid $x$ has $I$ grid points, which are automatically adjusted so that half of the grid points cover the negative part of the $(s, S)$ band and half of them cover the positive part. In this illustrative example $I = 5$ (we use a larger $I$ when solving the model). The adjustment is obtained by multiplying the auxiliary grid $x = [-1, -0.5, 0, 0.5, 1]$ by $\hat{s}_t(x)$: $x = x\hat{s}_t$

So far we have rewritten the law of motion of the firm distribution $g_t$. We now introduce the approximation we rely on for $g_t$. We approximate $g^c$ by a piece-wise linear function with equally spaced nodes $x_1, \ldots, x_I = -1, \ldots, 0, \ldots, 1$ with $g_t^c(x | x_i < x < x_{i+1}) \approx g_t^c(x_i) + \frac{x - x_i}{x_{i+1} - x_i} \frac{g_{t-1}^c(x_{i+1}) - g_{t-1}^c(x_i)}{x_{i+1} - x_i}$.

Note that the auxiliary grid for $x$ is exogenous. However, this exogenous auxiliary grid defines an *endogenous grid* for $x = \hat{s}_t x$, which, at each $t$, exactly spans the the $(s, S)$ band and has a node at 0. Figure A1 illustrates the use of linear interpolation with an endogenous grid as we apply it here.

From now on, $g_t^c$ denotes the piece-wise linear approximated function and $g_t^c(x_i < x < x_{i+1})$ denotes a linear piece of it. Thus, the functions are approximated as

$$g_t^c(x) = (1 - \lambda_t(x)) \left[ \sum_{i=1}^{I-1} \int_{x_i}^{x_{i+1}} \frac{\hat{s}_{t-1}(x')}{\sigma_t} g_{t-1}^c(x_i < x' < x_{i+1}) \phi\left(h(x', x)\right) \, dx' + \frac{1}{\sigma_t} g_{t-1}^0 \phi\left(h(0, x)\right) \right],$$

$$g_t^0 = 1 - \sum_{i=1}^{I-1} \int_{x_i}^{x_{i+1}} g_t^c(x_i < x < x_{i+1}) \hat{s}_t(x) dx.$$

Notice that, in these expressions, the integrands are continuous in the interval $x_i < x < x_{i+1}$ since $x$ and $x'$ are of constant sign.

Also note that the distribution function is 0 outside the (S,s) band. Our piecewise linear $g_t^c$ in fact is only defined over the range where the distribution has positive mass, that is for $x \in [-1, 1]$. This is computationally efficient.

Within this range $(1 - \lambda_t(x)) = 1$ so we can drop it from the expression above.

$$g_t^c(x) = \left[ \sum_{i=1}^{I-1} \int_{x_i}^{x_{i+1}} \frac{\hat{s}_{t-1}(x')}{\sigma_t} g_{t-1}^c(x_i < x' < x_{i+1}) \phi\left(h(x', x)\right) \, dx' + \frac{1}{\sigma_t} g_{t-1}^0 \phi\left(h(0, x)\right) \right]$$

### E.2.2. Other Aggregation Equations

The equilibrium conditions contain two further aggregation equations that contain the function $g(\cdot)$, for which we use the piece-wise linear approximation of $g^c(\cdot)$. Recall the aggregate price index and the labor market clearing condition

$$e^{p_t^*(\epsilon-1)} = \int e^{x(1-\epsilon)} g_t(x)dx,$$
$$N_t = \frac{C_t}{A_t} e^{p_t^*(-\epsilon)} \int e^{x(-\epsilon)} g_t(x)d(x) + \eta \int \lambda_t(x + p_t^* - \sigma_t \varepsilon_t - \pi_t^*) g_{t-1}(x)d(x)$$

which we approximate as follows, after the change of variable to $\mathsf{x}$,

$$e^{p_t^*(\epsilon-1)} = \sum_{i=1}^{I-1} \int_{\mathsf{x}_i}^{\mathsf{x}_{i+1}} e^{\mathsf{x}(1-\epsilon)} \mathsf{g}_t^c(\mathsf{x}_{i-1} < \mathsf{x} < \mathsf{x}_{i+1}) \hat{s}_t(\mathsf{x}) d\mathsf{x} + \mathsf{g}_t^0,$$

$$N_t = \frac{C_t}{A_t} e^{p^*(-\epsilon)} \sum_{i=1}^{I-1} \int_{\mathsf{x}_i}^{\mathsf{x}_{i+1}} \left( e^{\mathsf{x}(-\epsilon)} \mathsf{g}_t^c(\mathsf{x}_{i-1} < \mathsf{x} < \mathsf{x}_{i+1}) \hat{s}_t(\mathsf{x}) d\mathsf{x} + \mathsf{g}_{t-1}^0 \right) + \eta \mathsf{g}_{t-1}^0.$$

### E.2.3. Value Function

Recall the value function is

$$V_t(x) = \Pi_t(x) + \Lambda_{t,t+1} \int \left(1 - \lambda_{t+1}(x - \sigma_{t+1}\varepsilon - \pi_{t+1}^*)\right) V_{t+1}(x - \sigma_{t+1}\varepsilon - \pi_{t+1}^*) d\xi(\varepsilon)$$
$$+ \Lambda_{t,t+1} \left(V_{t+1}(0) - \eta w_{t+1}\right) \int \lambda_{t+1}(x - \sigma_{t+1}\varepsilon - \pi_{t+1}^*) d\xi(\varepsilon).$$

We now express it in terms of $\mathsf{x}$ with $\mathsf{V}_t(\mathsf{x}) \equiv V_t(\mathsf{x}\hat{s}_t)$:

$$\mathsf{V}_t(\mathsf{x}) = \Pi_t(\mathsf{x}) + \Lambda_{t,t+1} \int \left(1 - \lambda_{t+1}\left(\frac{\mathsf{x}\hat{s}_t + \sigma_t \varepsilon + \pi_t^*}{\hat{s}_{t-1}}\right)\right) \mathsf{V}_{t+1}\left(\frac{\mathsf{x}\hat{s}_t + \sigma_t \varepsilon + \pi_t^*}{\hat{s}_{t-1}}\right) d\xi(\varepsilon)$$
$$+ \Lambda_{t,t+1} \left(\mathsf{V}_{t+1}(0) - \eta w_{t+1}\right) \int \lambda_{t+1}\left(\frac{\mathsf{x}\hat{s}_t + \sigma_t \varepsilon + \pi_t^*}{\hat{s}_{t-1}}\right) d\xi(\varepsilon).$$

Note that by definition $\mathsf{V}_t(0) - \eta \frac{w_{t+1}}{A_{t+1}} = \mathsf{V}_t(-1) = \mathsf{V}_t(1)$ and $\mathsf{V}_t'(0) = 0$. The first two equalities are straightforward; the next subsection discusses the latter.

After the change of variable to $\mathsf{x}'$, which is analogous to the change of variable applied to $\mathsf{g}_t^c$ previously, we can rewrite $\mathsf{V}_t(\mathsf{x})$ as

$$\mathsf{V}_t(\mathsf{x}) = \Pi_t(\mathsf{x}) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int \hat{s}_{t+1}(\mathsf{x}') \left(1 - \lambda_{t+1}(\mathsf{x}')\right) \mathsf{V}_{t+1}(\mathsf{x}') \phi\left(h(\mathsf{x}, \mathsf{x}')\right) d\mathsf{x}'$$
$$+ \Lambda_{t,t+1} \left(\mathsf{V}_{t+1}(0) - \eta w_{t+1}\right) \int \hat{s}_{t+1}(\mathsf{x}') \lambda_{t+1}(\mathsf{x}') \phi\left(h(\mathsf{x}, \mathsf{x}')\right) \frac{1}{\sigma_{t+1}} d\mathsf{x}'$$

Since the price updating probability $\lambda_{t+1}(\mathsf{x}) = 1$ for any $\mathsf{x}$ outside the $(S, s)$ band, we can restrict the first integral to the range $[-1, 1]$. The last term in the second line (which captures the probability of updating a price tomorrow, given the current state) can be replaced by 1 minus the probability of not updating the price tomorrow. The latter is given by an integral over the range $[-1, 1]$. So we write:

$$\begin{aligned}\mathsf{V}_t(\mathsf{x}) =& \Pi_t(\mathsf{x}) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \left(1 - \lambda_{t+1}(\mathsf{x}')\right) \mathsf{V}_{t+1}(\mathsf{x}') \phi \left(h(\mathsf{x}, \mathsf{x}')\right) d\mathsf{x}' \\ &+ \Lambda_{t,t+1} \left(\mathsf{V}_{t+1}(0) - \eta w_{t+1}\right) \left(1 - \frac{1}{\sigma_{t+1}} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \left(1 - \lambda_{t+1}(\mathsf{x}')\right) \phi \left(h(\mathsf{x}, \mathsf{x}')\right) d\mathsf{x}'\right).\end{aligned}$$

In the inaction region, the price updating probability $\Lambda_{t,t+1}(\mathsf{x}) = 0$, so:

$$\begin{aligned}\mathsf{V}_t(\mathsf{x}) =& \Pi_t(\mathsf{x}) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \mathsf{V}_{t+1}(\mathsf{x}') \phi \left(h(\mathsf{x}, \mathsf{x}')\right) d\mathsf{x}' \\ &+ \Lambda_{t,t+1} \left(\mathsf{V}_{t+1}(0) - \eta w_{t+1}\right) \left(1 - \frac{1}{\sigma_{t+1}} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \phi \left(h(\mathsf{x}, \mathsf{x}')\right) d\mathsf{x}'\right).\end{aligned}$$

So far we have normalized the support of the value function. Additionally, it is convenient to normalize further the value function itself. We normalize the value function by its maximal value $\mathsf{V}_t(0)$, and denote the normalized value function by $\mathsf{v}_t(\mathsf{x})$: $\mathsf{v}_t(\mathsf{x}) \equiv \mathsf{V}_t(\mathsf{x}) - \mathsf{V}_t(0)$. The expression above can be re-written as:

$$\begin{aligned}&\mathsf{v}_t(\mathsf{x}) \equiv \mathsf{V}_t(\mathsf{x}) - \mathsf{V}_t(0) = \Pi_t(\mathsf{x}) - \Pi_t(0) \\ &+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left(\int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \left[\mathsf{V}_{t+1}(\mathsf{x}') \phi\left(\frac{\mathsf{x} - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right) - \mathsf{V}_{t+1}(\mathsf{x}') \phi\left(\frac{0 - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right)\right] d\mathsf{x}'\right) \\ &+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left(- \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \left[\phi\left(\frac{\mathsf{x} - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right) - \phi\left(\frac{0 - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right)\right] d\mathsf{x}'\right) \left(V_{t+1}(0) - \eta w_{t+1}\right) \\ &= \Pi_t(\mathsf{x}) - \Pi_t(0) \\ &+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left(\int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \left[\mathsf{v}_{t+1}(\mathsf{x}') \left(\phi\left(\frac{\mathsf{x} - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right) - \phi\left(\frac{0 - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right)\right)\right] d\mathsf{x}'\right) \\ &+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left(- \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \left[\phi\left(\frac{\mathsf{x} - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right) - \phi\left(\frac{0 - \mathsf{x}' - \pi^*_{t+1}}{\sigma_{t+1}}\right)\right] d\mathsf{x}'\right) \left(-\eta w_{t+1}\right)\end{aligned}$$

Following our approach for $\mathsf{g}^c(\cdot)$, we approximate $\mathsf{v}(\cdot)$ by a piece-wise linear function with nodes $\mathsf{x}_1, \ldots, \mathsf{x}_I = -1, \ldots, 0, \ldots, 1$ with $\mathsf{v}_t(\mathsf{x}|\mathsf{x}_i < \mathsf{x} < \mathsf{x}_{i+1}) \approx \mathsf{v}_t(\mathsf{x}_i) + \frac{\mathsf{x} - \mathsf{x}_i}{\mathsf{x}_{i+1} - \mathsf{x}_i} \frac{\mathsf{v}_t(\mathsf{x}_{i+1}) - \mathsf{v}_t(\mathsf{x}_i)}{\mathsf{x}_{i+1} - \mathsf{x}_i}$.

From now on, $\mathsf{v}_t$ denotes the piece-wise linear approximated function and $\mathsf{v}_t(\mathsf{x}_i < \mathsf{x} < \mathsf{x}_{i+1})$ denotes a linear piece of it. Thus, this function $\mathsf{v}_t(\mathsf{x})$ is approximated as

$$\begin{aligned}\mathsf{v}_t(\mathsf{x}) =& \Pi_t(\mathsf{x}) - \Pi_t(0) \\ &+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \sum_{i=1}^{I-1} \int_{\mathsf{x}_i}^{\mathsf{x}_{i+1}} \hat{s}_{t+1}(\mathsf{x}') \mathsf{v}_{t+1}(\mathsf{x}_i < \mathsf{x}' < \mathsf{x}_{i+1}) (\phi\left(h(\mathsf{x}, \mathsf{x}')\right) - \phi\left(h(0, \mathsf{x}')\right)) d\mathsf{x}' \\ &+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left(-\eta w_{t+1}\right) \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') (\phi\left(h(\mathsf{x}, \mathsf{x}')\right) - \phi\left(h(0, \mathsf{x}')\right)) d\mathsf{x}'.\end{aligned}$$

#### E.2.4. Optimality condition for reset price

We proceed in the same way for the derivative of the value function. We start with

$$0 = V_t'(0) = \Pi_t'(0) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \left. \frac{\partial \phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x} \right|_{x=0} dx'$$
$$+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left( \phi\left(\frac{-S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) \right) (V_{t+1}(0) - \eta w_{t+1})$$

where

$$\left. \frac{\partial \phi\left(\frac{x-x'-\pi_{t+1}^*}{\sigma_{t+1}}\right)}{\partial x} \right|_{x=0} = \frac{1}{\sqrt{2\pi}\sigma_{t+1}} \frac{-\pi_{t+1}^* - x'}{\sigma_{t+1}} e^{-\frac{1}{2}\left(\frac{-\pi_{t+1}^*-x'}{\sigma_{t+1}}\right)^2},$$
$$= \frac{\phi\left(\frac{-\pi_{t+1}^*-x'}{\sigma_{t+1}}\right)}{\sigma_{t+1}} \frac{-\pi_{t+1}^* - x'}{\sigma_{t+1}}$$

After change of variable to $\mathsf{x}$, this expression becomes

$$0 = \Pi_t'(0) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \mathsf{V}_{t+1}(\mathsf{x}') h(0, \mathsf{x}') \frac{\phi\left(h(0, \mathsf{x}')\right)}{\sigma_{t+1}} d\mathsf{x}'$$
$$+ \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left( \phi\left(\frac{-S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) \right) (\mathsf{V}_{t+1}(0) - \eta w_{t+1}).$$

Now we re-express this in terms of $\mathsf{v}(\mathsf{x})$ using $\mathsf{V}_t(\mathsf{x}) = \mathsf{v}_t(\mathsf{x}) + \mathsf{V}_t(0)$ first, and the rearranging

$$\begin{aligned}
0 &= \Pi_t'(x) + \Lambda_{t,t+1} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \left(\mathsf{v}_{t+1}(\mathsf{x}') + \mathsf{V}_{t+1}(0)\right) h(0, \mathsf{x}') \frac{\phi\left(h(0, \mathsf{x}')\right)}{\sigma_{t+1}} d\mathsf{x}' \\
&\quad + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left( \phi\left(\frac{-S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) \right) (\mathsf{V}_{t+1}(0) - \eta w_{t+1}) \\
&= \Pi_t'(x) + \Lambda_{t,t+1} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \mathsf{v}_{t+1}(x') h(0, \mathsf{x}') \frac{\phi\left(h(0, \mathsf{x}')\right)}{\sigma_{t+1}} d\mathsf{x}' \\
&\quad + \Lambda_{t,t+1} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') h(0, \mathsf{x}') \frac{\phi\left(h(0, \mathsf{x}')\right)}{\sigma_{t+1}} dx' \mathsf{V}_{t+1}(0) \\
&\quad + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left( \phi\left(\frac{-S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) \right) (\mathsf{V}_{t+1}(0) - \eta w_{t+1}) \\
&= \Pi_t'(x) + \Lambda_{t,t+1} \int_{-1}^{1} \hat{s}_{t+1}(\mathsf{x}') \mathsf{v}_{t+1}(x') h(0, \mathsf{x}') \frac{\phi\left(h(0, \mathsf{x}')\right)}{\sigma_{t+1}} d\mathsf{x}' \\
&\quad - \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left( \phi\left(\frac{-S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}}\right) \right) \mathsf{V}_{t+1}(0)
\end{aligned}$$

$$+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)-\phi\left(\frac{-s_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)\right)(\mathrm{V}_{t+1}(0)-\eta w_{t+1})$$

$$= \Pi'_t(0)+\Lambda_{t,t+1}\int_{-1}^{1}\hat{s}_{t+1}(\mathrm{x}')\mathrm{v}_{t+1}(\mathrm{x}')h(0,\mathrm{x}')\frac{\phi\left(h(0,\mathrm{x}')\right)}{\sigma_{t+1}}d\mathrm{x}'$$

$$+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)-\phi\left(\frac{-s_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)\right)(-\eta w_{t+1})$$

and apply the piece-wise linear approximation of $\mathrm{v}(\mathrm{x})$:

$$0=\Pi'_t(0)+\Lambda_{t,t+1}\sum_{i=1}^{I-1}\int_{-1}^{1}\hat{s}_{t+1}(\mathrm{x}')\mathrm{v}_{t+1}(\mathrm{x_i}<\mathrm{x}'<\mathrm{x_{i+1}})h(0,\mathrm{x}')\frac{\phi\left(h(0,\mathrm{x}')\right)}{\sigma_{t+1}}d\mathrm{x}'$$
$$+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)-\phi\left(\frac{-s_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)\right)(-\eta w_{t+1}).$$

### E.3. Solving for Integrals

Let us collect the approximated equations we defined so far.

(A16)

$$\mathrm{v}_t(\mathrm{x})=\Pi_t(\mathrm{x})-\Pi_t(0)$$
$$+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\sum_{i=1}^{I-1}\int_{\mathrm{x}_i}^{\mathrm{x}_{i+1}}\hat{s}_{t+1}(\mathrm{x}')\mathrm{v}_{t+1}(\mathrm{x}_i<\mathrm{x}'<\mathrm{x}_{i+1})(\phi\left(h(\mathrm{x},\mathrm{x}')\right)-\phi\left(h(0,\mathrm{x}')\right))d\mathrm{x}'$$
$$+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}(-\eta w_{t+1})\int_{-1}^{1}\hat{s}_{t+1}(\mathrm{x}')(\phi\left(h(\mathrm{x},\mathrm{x}')\right)-\phi\left(h(0,\mathrm{x}')\right))d\mathrm{x}',$$

(A17)

$$0=\Pi'_t(0)+\Lambda_{t+1}\int_{-1}^{1}\hat{s}_{t+1}\mathrm{v}_{t+1}(\mathrm{x}')h(0,\mathrm{x}')\frac{\phi\left(h(0,\mathrm{x}')\right)}{\sigma_{t+1}}d\mathrm{x}'$$
$$+\frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)-\phi\left(\frac{-s_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)\right)(-\eta w_{t+1}),$$

(A18) $$\mathrm{g}^c_t(\mathrm{x})=\sum_{i=1}^{I-1}\int_{\mathrm{x}_i}^{\mathrm{x}_{i+1}}\frac{\hat{s}_{t-1}(\mathrm{x}')}{\sigma_t}\mathrm{g}^c_{t-1}(\mathrm{x}_i<\mathrm{x}'<\mathrm{x}_{i+1})\phi\left(h(\mathrm{x}',\mathrm{x})\right)\,d\mathrm{x}'+\frac{1}{\sigma_t}\mathrm{g}^0_{t-1}\phi\left(h(0,\mathrm{x})\right),$$

(A19) $$\mathrm{g}^0_t=1-\sum_{i=1}^{I-1}\int_{\mathrm{x}_i}^{\mathrm{x}_{i+1}}\mathrm{g}^c_t(\mathrm{x}_i<\mathrm{x}'<\mathrm{x}_{i+1})\hat{s}_t(\mathrm{x})d\mathrm{x},$$

(A20) $$e^{p^*_t(\epsilon-1)}=\sum_{i=1}^{I-1}\int_{\mathrm{x}_i}^{\mathrm{x}_{i+1}}e^{(\mathrm{x})(1-\epsilon)}\mathrm{g}^c_t(\mathrm{x}_i<\mathrm{x}'<\mathrm{x}_{i+1})\hat{s}_t(\mathrm{x})d\mathrm{x}+\mathrm{g}^0_t,$$

$$\text{(A21)} \qquad N_t = \frac{C_t}{A_t} e^{p_t^*(-\epsilon)} \left( \sum_{i=1}^{I-1} \int_{\mathrm{x}_i}^{\mathrm{x}_{i+1}} e^{\mathrm{x}(-\epsilon)} \mathrm{g}_t^c(\mathrm{x}_{i-1} < \mathrm{x} < \mathrm{x}_{i+1}) \hat{\mathrm{s}}_t(\mathrm{x}) d\mathrm{x} + \mathrm{g}_{t-1}^0 \right) + \eta \mathrm{g}_{t-1}^0.$$

The integrals in all of these expressions can be computed analytically, since the integrands consist of affine functions multiplied by expressions that have closed form anti-derivatives. Figure A2 illustrates this graphically for the integral in the equation for $\mathrm{g}_t^c(x)$ (A18).

We now determine the solution of those integrals, equation by equation. Given the coefficients of the affine functions, which depend on the values of $\mathrm{v}_{t+1}(\mathrm{g}_{t-1})$ at the grid points $\mathrm{x}_i$, we can then write the solutions as a function that is linear in the elements of the vector $\mathrm{v}_{t+1}(\mathrm{x}_i)$ $(\mathrm{g}_{t-1}(\mathrm{x}_i))$. We now explain this for the simple case of the integral in equation A19. The other equations require some more tedious algebra, which we conveniently executed using symbolic math and which we omit here for brevity, but are conceptually equivalent.

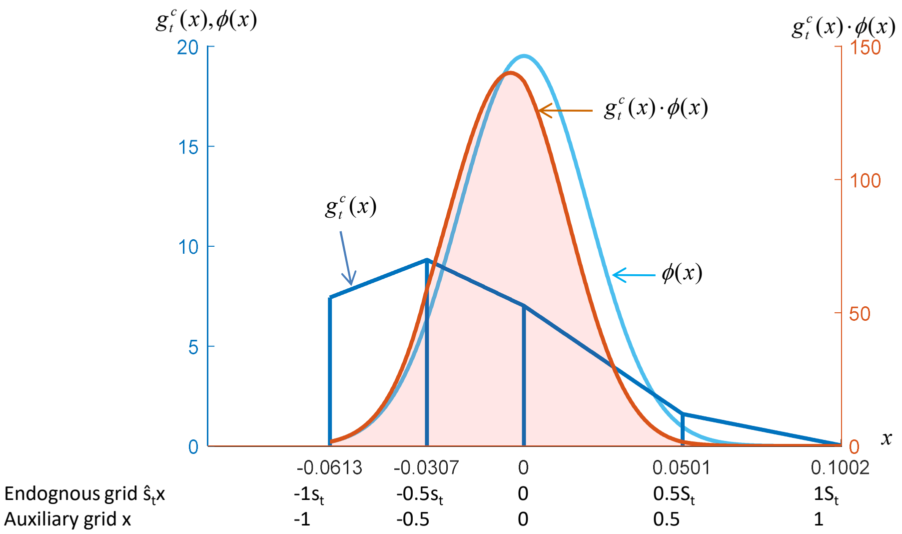


FIGURE A2. This figure schematically explains the analytical evaluation of integrals, given the linear approximation of the distribution and value functions. It shows the piece-wise linearly approximated distribution $\mathrm{g}_t^c(x)$ in blue, the normal pdf $\phi(x)$ in light blue and the product of the two $\mathrm{g}_t^c(x)\phi(x)$ in orange, where $x = \mathrm{x}\hat{\mathrm{s}}_t$. The orange area thus corresponds to the term $\sum_{i=1}^{I-1} \int_{\mathrm{x}_i}^{\mathrm{x}_{i+1}} \frac{\hat{\mathrm{s}}_{t-1}(\mathrm{x}')}{\sigma} \mathrm{g}_{t-1}^c(\mathrm{x}_i < \mathrm{x}' < \mathrm{x}_{i+1}) \phi\left(h(\mathrm{x}', \mathrm{x})\right) d\mathrm{x}'$ in equation (A18).

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### E.3.1. Mass Point

The integral over an affine function $f(x)$ from $x_1$ to $x_2$ is given by

$$\int_{x_1}^{x_2} f(x)\, dx = \frac{(f(x_1) + f(x_2))}{2}(x_2 - x_1)$$

thus

$$\sum_{i=1}^{I-1} \int_{x_i}^{x_{i+1}} f(x)\, dx = \sum_{i=1}^{I-1} \frac{(f(x_i) + f(x_{i+1}))}{2}(x_{i+1} - x_i).$$

Collecting the common terms on the right-hand side we get

$$\sum_{i=1}^{I-1} \int_{x_i}^{x_{i+1}} f(x)\, dx = \frac{\Delta x}{2}\left(f(x_1) + 2\sum_{i=2}^{I-1} f(x_i) + f(x_I)\right).$$

Applying this formula to equation (A19), which defines the mass point at $x = 0$, and re-arranging terms we get

$$\text{(A22)} \qquad g_t^0 = 1 - \mathbf{e}_t^T \mathbf{g}_t^c$$

where $\mathbf{e}_t^T = [0.5,\ 1, \ldots, 1,\ 0.5]\Delta x$. Note that this formula corresponds to the trapezoid rule. The blue area in Figure A1 illustrates the application of the trapezoid rule.

#### E.3.2. Aggregate Price Index

By the same logic, the aggregate price index in (A20) is computed as

$$\text{(A23)} \qquad e^{p_t^*(\epsilon-1)} = \sum_{i=1}^{I}(g_t^c(x_i)\mathbb{1}_{i\neq 1} d_{t,i,i-1,1-\varepsilon} + g_t^c(x_i)\mathbb{1}_{i\neq I} d_{t,i,i+1,1-\varepsilon}) + g_t^0$$

where

$$d_{t,i,j,\varepsilon} = \frac{\left(e^{(\epsilon)x_i\hat{s}_{t,i}}\left((\epsilon)\left(x_i\hat{s}_{t,i} - x_j\hat{s}_{t,j}\right) - 1\right) + e^{(\epsilon)x_j\hat{s}_{t,j}}\right)}{(\epsilon)^2\left(x_i\hat{s}t, i - x_j\hat{s}_{t,j}\right)}$$

and where $\hat{s}_{t,i} \equiv \hat{s}_t(x_i)$ and where $\mathbb{1}_{i\neq 1}$ and $\mathbb{1}_{i\neq I}$ are indicator functions equal to 1 when $i$ is different from 1 or $I$, that is whenever $g_t^c(x_i)$ is evaluated at the boundaries of the $(S, s)$ band. It plays a similar role as the values 0.5 at the two extremes of the vector $\mathbf{e}_t^T$ above.

Hence, we can re-write equation (A23) in matrix form as

$$\text{(A24)} \qquad e^{p_t^*(\epsilon-1)} = \mathbf{d}_{t,1-\varepsilon}^T \mathbf{g}_t^c + g_t^0$$

where $\mathbf{g}_t^c$ is the vector collecting the values of the the distribution function $g_t^c$ at the grid points and where the vector $\mathbf{d}_{t,1-\varepsilon}$ is

$$\mathbf{d}_{t,1-\varepsilon} = \left[\mathbb{1}_{i\neq 1} d_{t,i,i-1,1-\varepsilon} + \mathbb{1}_{i\neq I} d_{t,i,i+1,1-\varepsilon}\right]_{i=1}^{I}.$$

Here we have adopted the notation that $[x_i]_{i=1}^{I}$ denotes a $I \times 1$ vector with elements $x_i$.

### E.3.3. Labor Market

Following the previous subsection, the labor market condition (A21) is computed as

$$N_t = \frac{C_t}{A_t} e^{p_t^*(-\epsilon)} \left( \sum_{i=1}^{I} (\mathrm{g}_t^c(\mathrm{x}_i) \mathbb{1}_{i\neq 1} d_{t,i,i-1,-\varepsilon} + \mathrm{g}_t^c(\mathrm{x}_i) \mathbb{1}_{i\neq I} d_{t,i,i+1,-\varepsilon}) + \mathrm{g}_{t-1}^0 \right) + \eta \mathrm{g}_{t-1}^0$$

which we re-write in matrix form as

$$N_t = \frac{C_t}{A_t} e^{p_t^*(-\epsilon)} \left( \mathbf{d}_{t,-\varepsilon}^T \mathbf{g}_t^c + \mathrm{g}_{t-1}^0 \right) + \eta \mathrm{g}_{t-1}^0. \tag{A25}$$

### E.3.4. Distribution

Once we have evaluated the integrals, the distribution function in (A18) can be written as:

$$\mathrm{g}_t^c(\mathrm{x}_j) = \sum_{i=1}^{I} \frac{1}{2\sqrt{2\pi}} \mathrm{g}_{t-1}^c(\mathrm{x}_i) \left[ \mathbb{1}_{i\neq 1} f_{t,i,i-1,j} + \mathbb{1}_{i\neq I} f_{t,i,i+1,j} \right] + \frac{1}{\sigma_t} \mathrm{g}_{t-1}^0 \phi \left( \frac{-\hat{s}_{t,j}\mathrm{x}_j - \pi_t^*}{\sigma_t} \right) \tag{A26}$$

where from now on, $\pi$ without time subindex, denotes the scalar $\pi$, $f_{t,i,\bar{i},j}$ and $\mathcal{P}_{t,i,j}$ are defined as

$$f_{t,i,\bar{i},j} = \frac{\sqrt{2\pi}\left(\mathcal{P}_{t,\bar{i},j}\right)\left(\mathrm{erf}\left(\frac{\mathcal{P}_{t,\bar{i},j}}{\sqrt{2}\sigma_t}\right) - \mathrm{erf}\left(\frac{\mathcal{P}_{t,i,j}}{\sqrt{2}\sigma_t}\right)\right) + 2\sigma_t\left(\exp\left(-\frac{\mathcal{P}_{t,\bar{i},j}{}^2}{2\sigma_t^2}\right) - \exp\left(-\frac{\mathcal{P}_{t,i,j}{}^2}{2\sigma_t^2}\right)\right)}{\left|\mathrm{x}_i\hat{s}_{t-1,i} - \mathrm{x}_{\bar{i}}\hat{s}_{t-1,\bar{i}}\right|},$$

$$\mathcal{P}_{t,i,j} = -\mathrm{x}_i\hat{s}_{t-1,i} + \mathrm{x}_j\hat{s}_{t,j} + \pi_t^*.$$

For compactness, define

$$\mathbf{g}_t^c \equiv \left[\mathrm{g}_t^c(\mathrm{x}_j)\right]_{j=1}^{I}$$

$$\mathbf{F}_t \equiv \left[\frac{1}{2\sqrt{2\pi}}\left(\mathbb{1}_{i\neq 1} f_{t,i,i-1,j} + \mathbb{1}_{i\neq I} f_{t,i,i+1,j}\right)\right]_{j=1,i=1}^{I,I}$$

$$\mathbf{f}_t \equiv \left[\frac{1}{\sigma_t}\phi\left(\frac{-\hat{s}_{t,j}\mathrm{x}_j - \pi_t^*}{\sigma_t}\right)\right]_{j=1}^{I}$$

where $\mathbf{g}_t^c$ and $\mathbf{f}_t$ are vectors with the probability mass function $\mathrm{g}_t^c$ and the scaled and shifted normal distribution at the grid points, respectively, $\mathbf{F}_t$ is a matrix that captures the idiosyncratic transitions due to firm-level quality shocks and where we have adopted the notation that $\left[x_{i,j}\right]_{j=1,i=1}^{J,I}$ denotes a $J \times I$ matrix with elements $x_{j,i}$. Thus, equation A26 can be represented in matrix form as

(A27) $$\mathbf{g}_{\boldsymbol{t}}^{\boldsymbol{c}} = \mathbf{F}_t \mathbf{g}_{\boldsymbol{t-1}}^{\boldsymbol{c}} + \mathbf{f}_t g_{t-1}^0.$$

### E.3.5. Value function

Once we have evaluated the integrals, and denoting the standard normal cdf by $\Phi(\cdot)$ and the central grid point by $i_0$ (i.e. for $x_{i_0} = 0$), the value function A16 can be written as

(A28)
$$\begin{aligned} v_t(x_j) &= \Pi_{j,t} - \Pi_{j,t}(0) \\ &+ \Lambda_{t,t+1} \sum_{i=1}^{I} \frac{1}{2\sqrt{2\pi}} v_{t+1}(x_i) \left( \mathbb{1}_{i\neq 1}(a_{t,i,i-1,j} - a_{t,i_0,i_0-1,j}) + \mathbb{1}_{i\neq I}(a_{t,i,i+1,j} - a_{t,i_0,i_0+1,j}) \right) \\ &+ \Lambda_{t,t+1} (-\eta w_{t+1}) \left( \Phi\left(\frac{\mathcal{P}_{t+1,j,I}}{\sigma_{t+1}}\right) - \Phi\left(\frac{\mathcal{P}_{t+1,j,1}}{\sigma_{t+1}}\right) - \Phi\left(\frac{\mathcal{P}_{t+1,i_0,I}}{\sigma_{t+1}}\right) + \Phi\left(\frac{\mathcal{P}_{t+1,i_0,1}}{\sigma_{t+1}}\right) \right) \end{aligned}$$

where

(A29)
$$a_{t,i,\bar{i},j} = \frac{\sqrt{2\pi}\left(\mathcal{P}_{t+1,j,\bar{i}}\right)\left(\operatorname{erf}\left(\frac{\mathcal{P}_{t+1,j,\bar{i}}}{\sqrt{2}\sigma_{t+1}}\right) - \operatorname{erf}\left(\frac{\mathcal{P}_{t+1,j,i}}{\sqrt{2}\sigma_{t+1}}\right)\right) + 2\sigma_{t+1}\left(\exp\left(-\frac{\left(\mathcal{P}_{t+1,j,\bar{i}}\right)^2}{2\sigma_{t+1}^2}\right) - \exp\left(-\frac{\left(\mathcal{P}_{t+1,j,i}\right)^2}{2\sigma_{t+1}^2}\right)\right)}{\left| x_i \hat{s}_{t+1,i} - x_{\bar{i}} \hat{s}_{t+1,\bar{i}} \right|}.$$

For compactness, let us define

$$\begin{aligned} \mathbf{v}_t &\equiv \left[ v_t(x_j) \right]_{j=1}^{I}, \\ \Pi_t &\equiv \left[ \Pi_{j,t} - \Pi_{j,t}(0) \right]_{j=1}^{I}, \\ \mathbf{A}_t &\equiv \left[ \Lambda_{t,t+1} \frac{1}{2\sqrt{2\pi}} (\mathbb{1}_{i\neq 1}(a_{t,i,i-1,j} - a_{t,i_0,i_0-1,j}) + \mathbb{1}_{i\neq I}(a_{t,i,i+1,j} - a_{t,i_0,i_0+1,j})) \right]_{j=1,i=1}^{I,I}, \\ \mathbf{b}_{t+1} &\equiv \left[ \Lambda_{t,t+1} \left( \Phi\left(\frac{\mathcal{P}_{t+1,j,I}}{\sigma_{t+1}}\right) - \Phi\left(\frac{\mathcal{P}_{t+1,j,1}}{\sigma_{t+1}}\right) - \Phi\left(\frac{\mathcal{P}_{t+1,i_0,I}}{\sigma_{t+1}}\right) + \Phi\left(\frac{\mathcal{P}_{t+1,i_0,1}}{\sigma_{t+1}}\right) \right) \right]_{j=1}^{I} \end{aligned}$$

where $\mathbf{v}_t$ and $\mathbf{b}_{t+1}$ are vectors that evaluate the value function $v_t$ and the adjustment probability at different grid points, $\Pi_t$ is the vector of profit differences, while $\mathbf{A}_t$ is a matrix that represents the idiosyncratic transition due to firm-level quality shocks and price updating. Thus, equation (A28) can be represented in matrix form as

(A30) $$\mathbf{v}_t = \Pi_t + [\mathbf{A}_t \mathbf{v}_{t+1} - \mathbf{b}_{t+1} \eta w_{t+1}].$$

### E.3.6. Optimality condition for reset price

After evaluating the integral, we can write the optimality condition in (A17) as

$$0 = \Pi_t'(0) + \Lambda_{t,t+1}\sum_{i=1}^{I} \mathrm{v}_{t+1}(\mathrm{x_i})\frac{1}{2}\left(\mathbb{1}_{i\neq 1}c_{t,i,i-1,i_0} + \mathbb{1}_{i\neq I}c_{t,i,i+1,i_0}\right) + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)\right)(-\eta w_{t+1}) \tag{A31}$$

where

$$c_{t,i,\bar{i},j} = \frac{\mathrm{erf}\left(\frac{\mathcal{P}_{t+1,j,i}}{\sqrt{2}\sigma}\right) - \mathrm{erf}\left(\frac{\mathcal{P}_{t+1,j,\bar{i}}}{\sqrt{2}\sigma}\right)}{x_i\hat{s}_{t+1,i} - x_{\bar{i}}\hat{s}_{t+1,\bar{i}}} - \frac{\sqrt{\frac{2}{\pi}}\exp\left(-\frac{(\mathcal{P}_{t+1,j,i})^2}{2\sigma^2}\right)}{\sigma}. \tag{A32}$$

We can write this equation using matrix notation:

$$0 = \Pi_t'(0) + \mathbf{c}_{t+1}^T\mathbf{v}_{t+1} + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)\right)(-\eta w_{t+1}) \tag{A33}$$

where

$$\mathbf{c}_{t+1} = \left[\Lambda_{t,t+1}\frac{1}{2}\left(\mathbb{1}_{i\neq 1}c_{t,i,i-1,i_0} + \mathbb{1}_{i\neq I}c_{t,i,i+1,i_0}\right)\right]_{i=1}^{I}. \tag{A34}$$

### E.4. Final equation system

Collecting the thus derived equations, and combining them with the remainder of the private equilibrium conditions (which contain no infinite dimensional objects) and the objective, we can approximate the infinite dimensional planner's problem by the following finite dimensional planner's problem

$$\max_{\{\mathbf{g_t^c}, g_t^0, \mathbf{v}_t, C_t, w_t, p_t^*, s_t, S_t, \pi_t^*\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \upsilon\left(\frac{C_t}{A_t}e^{p_t^*(-\epsilon)}\left(\mathbf{d}_{t,-\varepsilon}^T\mathbf{g_t^c} + g_{t-1}^0\right) + \eta g_{t-1}^0\right)\right)$$

subject to

$$\begin{aligned}
w_t &= \upsilon C_t^{\gamma}, \\
\mathbf{v}_t &= \Pi_t + \mathbf{A}_t\mathbf{v}_{t+1} - \mathbf{b}_{t+1}\eta w_{t+1}, \\
\mathbf{v}_{t,1} &= -\eta w_t, \\
\mathbf{v}_{t,I} &= -\eta w_t, \\
0 &= \Pi_t'(0) + \mathbf{c}_{t+1}^T\mathbf{v}_{t+1} + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}}\left(\phi\left(\frac{-S_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right) - \phi\left(\frac{-s_{t+1}-\pi^*_{t+1}}{\sigma_{t+1}}\right)\right)(-\eta w_{t+1}), \\
\mathbf{g_t^c} &= \mathbf{F}_t\mathbf{g_{t-1}^c} + \mathbf{f}_t g_{t-1}^0,
\end{aligned}$$

$$\begin{aligned}
g_t^0 &= 1 - \mathbf{e}_t^T \mathbf{g_t^c}, \\
e^{p_t^*(\epsilon-1)} &= \mathbf{d}_{t,1-\varepsilon}^T \mathbf{g_t^c} + g_t^0.
\end{aligned}$$

Here the choice variables $\mathbf{v}_t$ and $\mathbf{g_t^c}$ are vectors of length $I$. The rest of the choice variables are scalars. Note that the choice variables $p_t^*, s_t, S_t, \pi_t^*$ implicitly appear in the problem (inside the vectors and matrices $\mathbf{A}_t$, $\mathbf{b}_t$, etc.)

As already explained at the beginning of this Appendix, we solve for the FOCs of this system by symbolic differentiation. The resulting system of FOCs is then solved in the sequence space. We next explain how we find the steady state, which serves as initial and terminal condition for dynamic simulations.

### E.5. Steady state

To solve for the steady state of the private equilibrium conditions, given a policy $\bar{\pi}$, the algorithm is as follows. We rely on steady-state relationships $w = \upsilon C^\gamma$, and $R = (1+\pi)/\beta$ and $\pi = \pi^*$. We start with a guess for the real wage $w$, the optimal rest price $p^*$, and the boundaries of the $(S, s)$ band $s$ and $S$ then:

a. Compute consumption $C = \left(\frac{w}{\upsilon}\right)^{1/\gamma}$.
b. Using $\pi = \pi^* = \bar{\pi}$, $C$ and the 4 initial guesses, solve for that stationary value function using the Bellman equation and the stationary distribution using the law of motion of the distribution. Both have closed form solutions given the guesses.

$$\begin{aligned}
\mathbf{v} &= (\mathbf{I} - \mathbf{A})^{-1} (\Pi - \mathbf{b}\eta w), \\
\mathbf{g}^c &= \left(\mathbf{I} - \mathbf{F} + \mathbf{f}\mathbf{e}^T\right)^{-1} \mathbf{f}, \\
g^0 &= 1 - \mathbf{e}^T \mathbf{g}^c
\end{aligned}$$

c. Compute the residuals of the 4 remaining equations

$$\begin{aligned}
\mathbf{v}_{t,1} &= -\eta w_t, \\
\mathbf{v}_{t,I} &= -\eta w_t, \\
0 &= \Pi_t'(0) + \mathbf{c}_{t+1}^T \mathbf{v}_{t+1} + \frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \left( \phi\left( \frac{-S_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}} \right) - \phi\left( \frac{-s_{t+1} - \pi_{t+1}^*}{\sigma_{t+1}} \right) \right) (-\eta w_{t+1}), \\
e^{p_t^*(\epsilon-1)} &= \mathbf{h}_{t,1-\varepsilon}^T \mathbf{g_t^c} + g_t^0.
\end{aligned}$$

d. Use a Newton method to update the 4 guesses ($w$, $p^*$, $s$, $S$) and return to step 1, until convergence of the residuals.

## Appendix F. The simplified model

***Model description.*** This static model version can be seen as a particular case of the complete model in which we set $\beta = 0$ and assume that the initial distribution is such that all firms have set the same price last period ($g_{-1}^c(x) = 0$, $g_{-1}^0 = 1$, $p_{-1}^* = 1$).

The model economy is then described by the following seven equations. (i) Firms' reset price maximizes their current profits

$$e^{p^*} = \frac{\epsilon}{(\epsilon-1)}(1-\tau)w. \tag{A35}$$

(ii-iii) Firms' price adjustment decision is characterized by the threshold values which equate the current profits in the case of no change in the price $p$, $\Pi(p)$, with profits under the optimal price $p^*$ minus the menu cost, $\Pi\left(e^{p^*}\right) - \eta w$. Using equation (A35) and Descartes' Rule of Signs, there are exactly 2 positive real roots, $s < S$.

$$\left((e^{p^*})^{1-\epsilon} - (1-\tau)w(e^{p^*})^{-\epsilon}\right) - \eta = \left(s^{1-\varepsilon} - (1-\tau)ws^{-\epsilon}\right), \tag{A36}$$

$$\left((e^{p^*})^{1-\epsilon} - (1-\tau)w(e^{p^*})^{-\epsilon}\right) - \eta = \left(S^{1-\varepsilon} - (1-\tau)wS^{-\epsilon}\right), \tag{A37}$$

The rest of the equations are as in the full model. They are given by the households labor supply, the frequency of price changes, the labor market clearing condition and the definition of the price level:

$$C = w, \tag{A38}$$

$$g_0 = 1 - \int_s^S g^c(p+\pi)dp, \tag{A39}$$

$$1 = \int_s^S e^{(p)(1-\epsilon)} g^c(p)\,dp + g^0 e^{p^*(1-\epsilon)}, \tag{A40}$$

$$N = C\left(\int_s^S e^{(p)(-\epsilon)} g^c(p)\,dp + g^0 e^{p^*(-\epsilon)}\right) + \eta g^0. \tag{A41}$$

Given the initial conditions, the price distribution is now normal with mean $\pi$ and variance $\sigma^2$: $g^c(p) = \phi\left(\frac{p+\pi}{\sigma}\right)$.

These equations define an equilibrium in 8 variables $w, \pi, C, N, s, S, g_0, p^*$. The policy maker has one degree of freedom to choose $\pi$. We calibrate the simple model following the same strategy as in the full model.

***Phillips curve***. Combining the first six equations to eliminate $w, s, S, g_0, p^*$, we can summarize the private equilibrium in this economy by one single equation in inflation and consumption:

$$1 = \left[\int_{\mathrm{s}(C,\tau)}^{\mathrm{S}(C,\tau)} e^{(p)(1-\epsilon)} \phi\left(\frac{p+\pi}{\sigma}\right) dp + \left(\frac{\varepsilon(1-\tau)}{\varepsilon-1}C\right)^{1-\epsilon}\left[1 - \int_{\mathrm{s}(C,\tau)}^{\mathrm{S}(C,\tau)} g^c(p+\pi)dx\right]\right] \tag{A42}$$

where $\mathrm{s}(C,\tau)$ and $\mathrm{S}(C,\tau)$ are implicit functions solving

$$\left(\frac{\epsilon}{\epsilon-1}(1-\tau)C\right)^{1-\epsilon} - ((1-\tau)C)^{1-\epsilon} - \eta = S^{1-\epsilon} - (1-\tau)CS^{-\epsilon}, \tag{A43}$$

$$\text{(A44)} \qquad \left(\frac{\epsilon}{\epsilon-1}(1-\tau)C\right)^{1-\epsilon} - ((1-\tau)C)^{1-\epsilon} - \eta = s^{1-\epsilon} - (1-\tau)CS^{-\epsilon}$$

for which an analytical solution exists if $\epsilon = 2$. This equation implicitly defines the Phillips curve, that is, a relationship between inflation and output (or, equivalently, the output gap).

***Welfare.*** Turning next to the planner's preferences, her objective is given by household utility $U = \log(C) - N$. Using the firms' conditions (A35), (A36), (A37), the definition of the price level (A40), the frequency (A39), and the labor-market clearing condition (A41), we can express the welfare function as:

$$\begin{aligned} U &= \log(C) - C\left(\int_{\mathsf{s}(\pi)}^{\mathsf{S}(\pi)} e^{(p)(-\epsilon)}\phi\left(\frac{p+\pi}{\sigma}\right)dp + \left(1 - \int_{\mathsf{s}(\pi)}^{\mathsf{S}(\pi)} \phi\left(\frac{p+\pi}{\sigma}\right)dx\right)e^{\mathsf{p}^*(\pi)(-\epsilon)}\right) \\ &\quad -\eta\left[1 - \int_{\mathsf{s}(\pi)}^{\mathsf{S}(\pi)} \phi\left(\frac{p+\pi}{\sigma}\right)dp\right] \end{aligned}$$

where where $\mathsf{s}(\pi)$, $\mathsf{S}(\pi)$ and $\mathsf{p}^*(\pi)$ are implicit functions solving the SS band conditions (A36), (A37), and the definition of the price level (A40). Just like the Phillips curve, this welfare function depends only on inflation and consumption. In the Calvo case without idiosyncratic shocks this representation of the welfare function, when approximated to second order, yields the well-known loss function $-\frac{1}{2}\left[\hat{c}^2 + \epsilon\left(\frac{1-\theta}{\theta}\right)\hat{\pi}^2\right]$ (see Galí 2008) where the 'hat' denotes deviation from the deterministic steady state.

In the menu cost model which we are interested in here, we can decompose the welfare gap relative to the efficient allocation ($C = N = 1$) in 3 terms:

$$\begin{aligned} U - U^{\text{eff}} &= \underbrace{log(C) - C - 1}_{\text{Average markup gap}} \\ &\quad \underbrace{-C\left(\int_{\mathsf{s}(\pi)}^{\mathsf{S}(\pi)} e^{(p)(-\epsilon)}\phi\left(\frac{p+\pi}{\sigma}\right)dp + \left(1 - \int_{\mathsf{s}(\pi)}^{\mathsf{S}(\pi)} \phi\left(\frac{p+\pi}{\sigma}\right)dx\right)e^{\mathsf{p}^*(\pi)(-\epsilon)} - 1\right)}_{\text{Price dispersion}} \\ &\quad \underbrace{-\eta\left(1 - \int_{\mathsf{s}(\pi)}^{\mathsf{S}(\pi)} \phi\left(\frac{p+\pi}{\sigma}\right)dp\right)}_{\text{Adjustment costs}} \\ &= \underbrace{-\log(1-\tau) - \overline{\mu} - \left(\frac{1}{e^{\overline{\mu}}(1-\tau)} - 1\right)}_{\text{Average markup gap}} - C\underbrace{\left(\zeta^{\mu(i)-\overline{\mu}} - 1\right)}_{\text{Price dispersion}} - \underbrace{\eta g_0}_{\text{Adjustment costs}}. \end{aligned}$$

## Appendix G. The CalvoPlus model

The setup follows very closely Section 2, so we introduce minimal modifications to notation. The menu cost now is a random variable $\tilde{\eta}$ such that

$$\tilde{\eta} = \begin{cases} \eta & \text{with prob } \alpha \\ 0 & \text{with prob } 1-\alpha \end{cases}$$

so the probability that a price $p$ is adjusted is

$$\Omega_t(p) = Pr\left[\tilde{\eta} = 0\right] + Pr\left[\tilde{\eta} = \eta\right]\lambda_t(p) = (1-\alpha) + \alpha\lambda_t(p).$$

The function $\lambda_t(p)$ is identical to the one in Section 2, but its interpretation is slightly different: it is the probability of a price change conditional on the menu cost being $\eta$. We reproduce it here for convenience:

$$\lambda_t\left(p\right) = 1\left[L\left(p\right) > 0\right]$$

where the difference in value between adjusting and not adjusting the price must be higher than the menu cost – which is expressed in terms of labor cost:

$$L\left(p\right) = \max_{p'} V_t\left(p'\right) - \eta w_t - V\left(p\right).$$

With that, the firm's value function now is

$$\begin{aligned} V_t\left(p\right) \;=\; & \Pi\left(p, w_t, A_t\right) \\ & + \alpha\mathbb{E}_t\left[\left(1-\lambda_{t+1}\left(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}\right)\right)\Lambda_{t,t+1}V_{t+1}\left(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}\right)\right] \\ & + \alpha\mathbb{E}_t\left[\lambda_{t+1}\left(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}\right)\Lambda_{t,t+1}\left(\max_{p'} V_{t+1}\left(p'\right) - \eta w_{t+1}\right)\right] \\ & + \left(1-\alpha\right)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\max_{p'} V_{t+1}\left(p'\right)\right)\right] \end{aligned}$$

which accounts for the fact that with probability $1-\alpha$ the price can be adjusted for free. As the menu cost is expressed in labor units, the labor market clearing condition in equation (20) in Section 2 must be modified to

$$N_t = \frac{C_t}{A_t}\int e^{p(-\epsilon)} g_t\left(p\right)dp + \alpha\eta\int \lambda_t\left(p-\sigma_t\varepsilon_t-\pi_t\right)g_{t-1}\left(p\right)dp$$

such that a share $\alpha$ of firms for which it is worthwhile to incur the menu cost $\eta$ actually pay it. Note that now the frequency of price changes is given by

$$f_t = \int \Phi_t\left(p\right)g_{t-1}\left(p\right)dp = (1-\alpha) + \alpha\int \lambda_t\left(p-\sigma_t\varepsilon_t-\pi_t\right)g_{t-1}\left(p\right)dp.$$

The next equation to modify is the law of motion of the price density function:

$$\begin{aligned} g_t(p) &= \alpha(1-\lambda_t(p)) \int g_{t-1}(p+\sigma_t\varepsilon_t+\pi_t)\, d\xi(\varepsilon) \\ &\quad +\delta\left(p-p_t^*\right) \int \left[(1-\alpha)+\alpha\lambda_t(\widetilde{p})\right]\left(\int g_{t-1}(\widetilde{p}+\sigma_t\varepsilon_t+\pi_t)\, d\xi(\varepsilon)\right) d\widetilde{p}. \end{aligned}$$

Summing up, the objective of the Ramsey problem in Section 3.1 now is

$$\max_{\substack{\{g_t^c(\cdot), g_t^0, V_t(\cdot), C_t, \\ w_t, p_t^*, s_t, S_t, \pi_t^*\}_{t=0}^{\infty}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \upsilon \frac{C_t}{A_t}\left(\int e^{\left(x+p_t^*\right)(-\epsilon_t)} g_t^c(p)\, dx + g_t^0 e^{\left(p_t^*\right)(-\epsilon)}\right) - \upsilon\eta[g_t^0-(1-\alpha)]\right)$$

subject to

$$\begin{aligned} w_t &= \nu C_t^{\gamma}, \\ V_t(x) &= \Pi\left(x, p_t^*, w_t, A_t\right) + \alpha\frac{\Lambda_{t,t+1}}{\sigma_{t+1}} \int_{s_t}^{S_t} \left[V_{t+1}\left(x'\right) \phi\left(\frac{\left(x-x'\right)-\pi_{t+1}^*}{\sigma_{t+1}}\right)\right] dx' + \\ &\quad +\alpha\Lambda_{t,t+1}\left(1-\frac{1}{\sigma_{t+1}}\int_{s_t}^{S_t} \phi\left(\frac{\left(x-x'\right)-\pi_{t+1}^*}{\sigma_{t+1}}\right) dx'\right)\left[V_{t+1}(0)-\eta w_{t+1}\right] \\ &\quad +(1-\alpha)V(0), \\ V_t(s_t) &= V_t(0)-\eta w_t, \\ V_t(S_t) &= V_t(0)-\eta w_t, \\ V_t'(0) &= 0, \\ g_t^c(x) &= \frac{\alpha}{\sigma_t}\int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1})\, \phi\left(\frac{(x_{-1}-x)-\pi_t^*}{\sigma_t}\right) dx_{-1} + \alpha g_{t-1}^0 \phi\left(\frac{-x-\pi_t^*}{\sigma_t}\right), \\ g_t^0 &= 1-\int_{s_t}^{S_t} g_t^c(x)\, dx, \\ 1 &= \int e^{\left(x+p_t^*\right)(1-\epsilon)} g_t^c(x)\, dx + g_t^0 e^{p_t^*(1-\epsilon)}. \end{aligned}$$