Optimal inflation with firm-level shocks[1]

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June 2023

[1]The views expressed here are those of the authors only and do not necessarily reflect the views of Deutsche Bundesbank, ECB or the Eurosystem.

Introduction

- Price setting models nowadays include *idiosyncratic shocks* to match empirical price change histograms (Golosov-Lucas 2007)
- These models (matching micro pricing moments) have been used widely for positive analysis, e.g. to infer the degree of monetary non-neutrality
- However, few papers attempt normative analysis; e.g. not much is known about the optimal rate of inflation in these models
- "Folk wisdom" for normative conclusions: firm-level shocks do not affect the optimality of zero inflation – the logic of the standard NK model applies

This project focuses on studying optimal inflation in canonical sticky price models with idiosyncratic productivity shocks a la Golosov-Lucas

We abstract from production networks and from shocks to market power

Preview: key friction

- Generally, efficiency requires $\frac{\text{reset price}(j)}{\text{price level}} = \frac{\text{aggregate productivity}}{\text{idiosyncratic productivity}(j)}$
- With price stickiness, reset prices are inefficiently forward-looking:

$$\frac{\text{reset price}(j)}{\text{price level}} = \frac{\text{aggregate productivity}}{\text{expected discounted idiosyncratic productivity}}$$

$\Longrightarrow$ reset price distribution compressed relative to flexible prices

Preview: reset price compression with AR(1) shocks

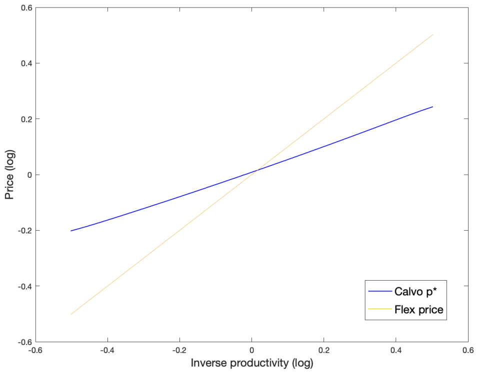


Figure: Optimal price compression in Calvo

Preview: nature of the misallocation

- Reset price of productive firms is inefficiently high, hence productive firms supply too small a share of aggregate demand relative to social optimum
- Reset price of unproductive firms is inefficiently low, hence unproductive firms supply too much of aggregate demand
- Reset price compression presents a new motive for choosing a *negative* optimal inflation rate $\pi^{\star}$
- Negative inflation rate helps align reset prices more closely with the current idiosyncratic shock realizations.
- By speeding up the pass-through of marginal cost shocks to prices, negative inflation increases aggregate productivity

Trade-off

Two forces:

1. The usual distortion between reset and non-reset prices (calling for zero inflation) in standard NK models
2. The distortion of reset prices themselves (calling for –100% inflation).

The first-best (efficient) allocation can no longer be achieved

The optimal trade-off is resolved at inflation rates around -2%.

For some plausible calibrations it may even be below the Friedman optimum (in a cashless economy!)

Related work

- Large literature on money non-neutrality in pricing models with idio shocks
  - Golosov-Lucas 2007: Near-neutrality of money
  - Midrigan 2011: Simple menu cost model matches poorly histogram of price changes
  - Karadi & Reiff 2019: Large tax shocks reveal that degree of money non-neutrality is very sensitive to shape of idiosyncratic shock distribution
  - Our results rely on shocks' mean reversion – a generic feature in the literature
- Small literature on $\pi^\star$ in pricing models with idiosyncratic shocks
  - Burstein & Hellwig 2008: Sticky prices vs Friedman rule
  - Blanco 2019: Sticky prices vs ZLB
  - Adam, Gautier, Santoro & Weber 2021: Sticky prices and product lifecycles
  - We focus on sticky prices only and rule out non-stationary productivity

Overview - NK model with firm-level shocks

- Representative household with discount rate $\beta \in (0, 1)$
- Aggregate output is CES composite with substitution elasticity θ
- Offset effect of flexible price markup using sales subsidy τ,

$$\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} = 1$$

$\implies$ flexible price allocation is efficient / first best

- Stochastic process for idiosyncratic productivity shocks
- Could have on top a non-stationary productivity shock common to all firms
- Aggregate productivity is endogenous and depends on inflation
- Calvo pricing, $\alpha \in [0, 1)$ denotes probability of not adjusting price

Technology of firm j and idiosyncratic productivity shocks

$$Y_{jt} Z_{jt} = L_{jt} \tag{1}$$

Inverse of idiosyncratic productivity shock is finite-state Markov

$$Z_{jt} = z' \xi_{jt} \tag{2}$$

- z is $K \times 1$ vector with inverse level of productivity in each state
- idio state ξ_{jt} equals $K \times 1$ unit vector e_j when idio state equals j
- unit unconditional mean $E(Z_{jt}) = 1$
- focus on ergodic distribution of idiosyncratic shocks, $\xi = E(\xi_{jt})$
- impose zero covariance btw idio shocks and aggregate variables

Aggregation of firms with idiosyncratic shocks

Aggregate technology

$$Y_t = \frac{1}{\Delta_t} L_t$$

Inverse aggregate productivity is output-weighted average of inverse idiosyncratic productivities,

$$\Delta_t = \int_0^1 Z_{jt} \underbrace{\left(\frac{Y_{jt}}{Y_t}\right)}_{=(P_{jt}/P_t)^{-\theta}} \text{dj}$$

Analytical aggregation yields recursive representation of Δ_t ▸ Details

Gross inflation $\Pi_t = \frac{P_t}{P_{t-1}}$ with P_t the welfare-based price level ▸ Details

Misallocation under zero inflation: distortion # 1

- WLOG let us assume shocks are i.i.d, 2 states (j, k) and zero inflation
- Output is demand determined. A firm that adjusted when its state was j and now has state k: $Z_k Y_j^* = L_k$
- Demand is determined by the reset price, which is a weighted average over current and expected future idiosyncratic costs $p_j^* = \vartheta w \left[(1 - \frac{\alpha}{2})Z_j + \frac{\alpha}{2} Z_k\right]$
- A fully flexible price instead is set as: $p_j^* = \vartheta w^e Z_j$
- So reset prices are distorted (compressed)

Misallocation under zero inflation: distortion #2

- Firms with constant prices have continuously evolving idiosyncratic costs
- Let us average labor over all firms with the same demand
  $Y_j^* \left[(1-\frac{\alpha}{2})Z_j + \frac{\alpha}{2}Z_k\right] = L^j$
- Under rational expectations the average j-firm has the cost level that the resetting firm anticipates when in state j
- Substituting in demand and the reset price, we obtain the contribution of the average j-firm to aggregate productivity:
  $\left[(1-\frac{\alpha}{2})Z_j + \frac{\alpha}{2}Z_k\right]^{(1-\theta)} = \frac{L^j}{L}\Delta^{(1-\theta)}$
- With flex prices instead, assuming same shares of firms in states j and k we obtain $(1-\frac{\alpha}{2})Z_j^{(1-\theta)} + \frac{\alpha}{2}Z_k^{(1-\theta)} = \frac{L^{ej}}{L^e}(\Delta^e)^{(1-\theta)}$
- With sticky prices aggregate productivity is influenced by prices that reflect averaged costs
- While with flex prices aggregate productivity is influenced by prices reflecting non-averaged idiosyncratic costs

Graphical illustration: zero inflation

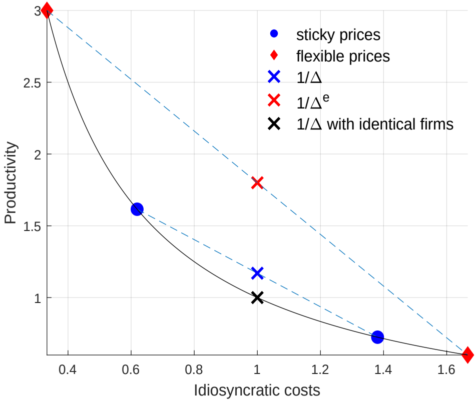


Figure: Zero inflation relative to flex price

Graphical illustration: negative inflation

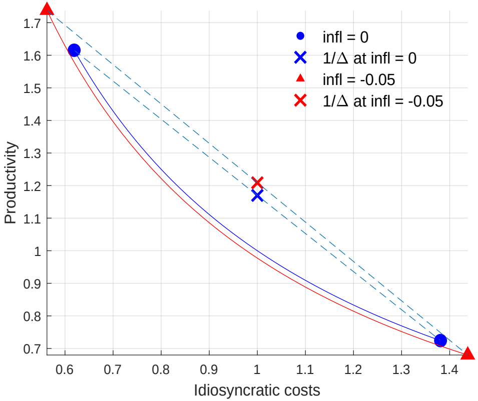


Figure: Negative inflation relative to zero inflation

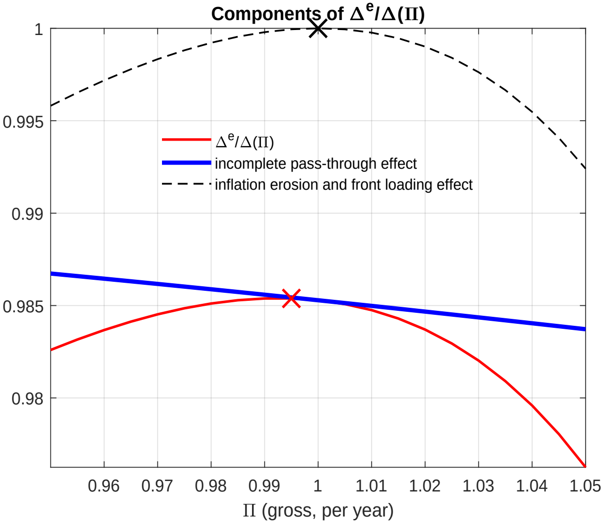
Decomposing the aggregate productivity distortion

Proposition: Consider the limit $\beta \to 1$ and idiosyncratic shocks that discretize a stationary AR(1) process with persistence $\rho < 1$ using Rouwenhorst (1995). Then, the productivity distortion is given by

$$\frac{\Delta^e}{\Delta(\Pi)} = \underbrace{\left(\frac{\sum_j \xi_j \left(1 + (z_j - 1)\right)^{1-\theta}}{\sum_j \xi_j \left(1 + (z_j - 1)\frac{1-\alpha\Pi^\theta}{1-\alpha\Pi^\theta\rho}\right)^{1-\theta}} \right)^{\frac{1}{1-\theta}}}_{\text{reset price compression (RPC)}} \cdot$$

$$\underbrace{\left(\frac{1-\alpha}{1-\alpha\Pi^{\theta-1}} \right)^{\frac{1}{\theta-1}}}_{\text{inflation erosion effect}} \cdot \underbrace{\left(\frac{1-\alpha\Pi^{\theta}}{1-\alpha\Pi^{\theta-1}} \right)}_{\text{inv front loading effect } = 1/\phi} .$$

With sticky prices, $\alpha > 0$, no inflation rate fully eliminates the productivity distortion.



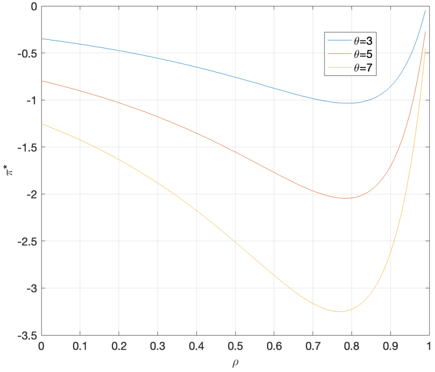
1. $\Delta^e/\Delta(\Pi) = 1$ *infeasible* given policy tradeoff from firm-level shocks
2. With firm-level shocks, prominent price stability result, $\Pi^\star = 1$, fails

Optimal inflation in Calvo

Consider a sticky-price steady state with firm-level productivity shocks, $\beta \to 1$ and $\frac{1}{1+\tau}\frac{\theta}{\theta-1} = 1$. Then, optimal inflation maximizing steady-state utility is negative,

$$\Pi^{\star} < 1,$$

and does not restore efficiency, $\Delta^e/\Delta(\Pi^{\star}) < 1$.



A generalized-hazard state-dependent pricing model

- Model by Woodford (2011). Has rational inattention microfoundations as shown by Steiner et al. (2017)
- Nests Calvo (1983) pricing and Golosov & Lucas (2007) with fixed menu costs as two polar cases

The equilibrium adjustment probability function takes the following form:

$$\lambda(G) = \frac{\bar{\lambda}\exp(\frac{G}{\xi})}{1-\bar{\lambda}+\bar{\lambda}\exp(\frac{G}{\xi})} \tag{3}$$

Parameter	Description	Value	Source
β	Monthly discount	0.9984	Annual real rate of 2%
γ	Intertemporal elast. of subst.	2	Golosov-Lucas (2007)
ζ	Frisch labor supply elast.	1	Ibid
χ	Coefficient on labor disutility	6	Ibid
θ	Elasticity of subst. across varieties	7	Ibid

Idiosyncratic shocks, exogenous and calibrated parameters

Logarithm of idiosyncratic productivity shocks follows

$$a_{jt} = \rho a_{jt-1} + \sigma \epsilon_t, \quad \epsilon_t \sim \mathrm{N}(0,1)$$

Shock process discretized into finite-state Markov process

Table: Estimated parameters

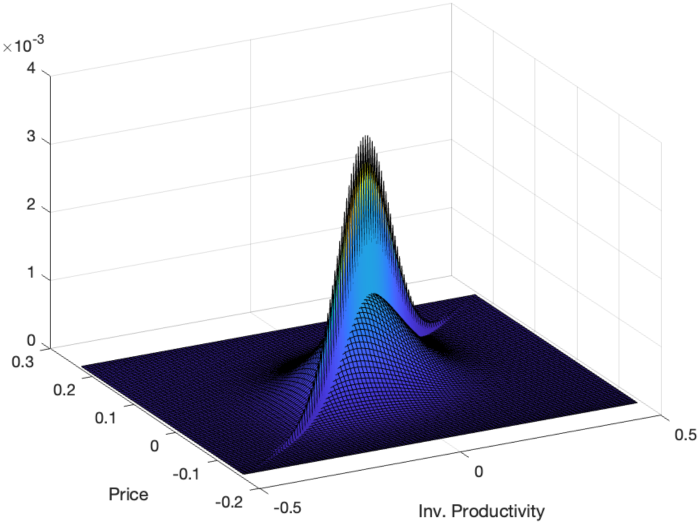
Description	Parameter	Value
Information cost	ξ	1.8370
Menu cost	κ	0.0359
Persistence of productivity shock	ρ	0.8989
Std dev of productivity shock	σ	0.0944

Matched moments

Table: Matched moments

Moment	Data	Model
Frequency of price changes	0.10	0.10
Std of price changes	0.0557	0.0557
Kurtosis of price changes	3.86	3.86
Persistence ρ_{adj}	0.896	0.896

Stationary distribution of firms



Adjustment probability function

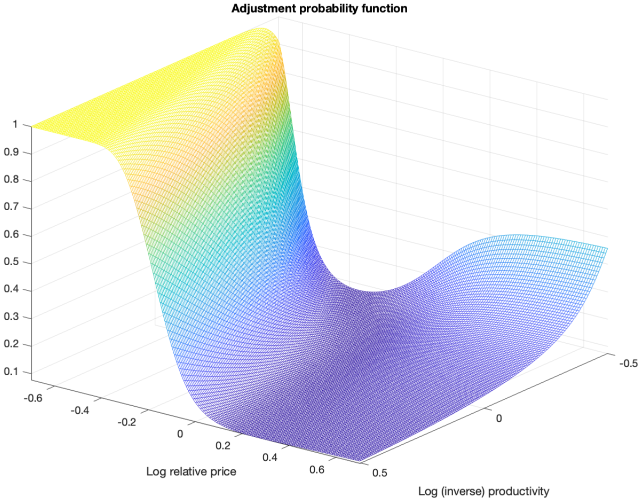


Figure: Adjustment probability function in SDP model

Utility

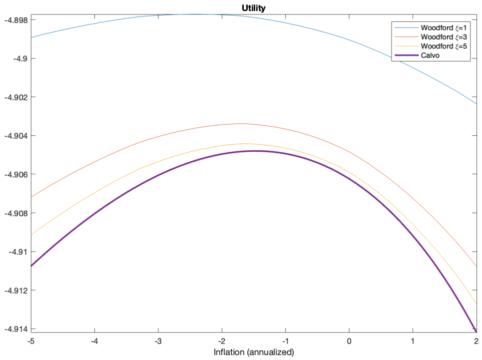


Figure: Inflation and utility in Calvo and Woodford models

- Calvo: $\pi^\star = -1.4\%$
- Woodford (2011): $\pi^\star = -2\%$
- 30-70 basis points productivity gain vis-a-vis targeting $+4\%$ inflation

Conclusions

Pricing models consistent with observed price change heterogeneity imply sizeable misallocation at zero inflation and so a role for monetary policy to reduce it

- Stickiness coupled with idiosyncratic productivity shocks distorts newly set prices thereby reducing aggregate productivity
- This distortion provides a new motive for choosing negative $\pi^\star$

Ongoing work

- Study Ramsey optimal policy, transitional dynamics, Ramsey steady state

Recursive representation of aggregate productivity

$$\Delta_t = z' V_t \tag{4}$$

$$V_t = (1-\alpha) D_t^{\theta} \text{diag}(z' N_t)^{-\theta} \xi + \alpha \Pi_t^{\theta} F V_{t-1} \tag{5}$$

$$N_t = I_K w_t + \alpha\beta F E_t[\Pi_{t+1}^{\theta} N_{t+1}] \tag{6}$$

$$D_t = 1 + \alpha\beta E_t[\Pi_{t+1}^{\theta-1} D_{t+1}] \tag{7}$$

- V_t is $K \times 1$, N_t is $K \times K$
- F is transition matrix of idiosyncratic shocks
- Π_t is gross inflation
- w_t is wage Back

Aggregation: Price level and recursive pricing rule

Price level

$$P_t^{1-\theta} = \int_0^1 P_{jt}^{1-\theta} \mathrm{dj} \tag{8}$$

Average reset price

$$(P_t^\star)^{1-\theta} \equiv \frac{1}{1-\alpha} \int_{J_t^\star} (P_{jt}^\star)^{1-\theta} \mathrm{dj}, \tag{9}$$

where set $J_t^\star$ has mass $1-\alpha$ and contains firms that can adjust price in t

$$\frac{P_{jt}^\star}{P_t} = \frac{\vartheta}{D_t} z' N_t \xi_{jt}, \tag{10}$$

$$(p_t^\star)^{1-\theta} = \left(\frac{\vartheta}{D_t}\right)^{1-\theta} z' N_t \ \mathrm{diag}(z' N_t)^{-\theta} \xi \tag{11}$$

with $\vartheta = \frac{\theta}{\theta-1} \frac{1}{1+\tau}$ ▸ Slides

Sketch of proof for optimal inflation being negative

$\Pi^\star$ maximizes steady state utility. With $\beta \to 1$ and $\frac{\theta}{\theta-1}\frac{1}{1+\tau} = 1$, obtain:

1. labor is independent of Π hence maximizing output also maximizes utility
2. $\delta(\Pi) = \mu(\Pi)^{-1}$ hence maximizing output requires maximizing $\delta(\Pi)$
3. $\delta(\Pi)' = 0$ implies that $\Pi^\star$ fulfills

$$\left(\frac{\Pi - 1}{\Pi}\right)\frac{1}{(1-\alpha\Pi^{\theta-1})(1-\alpha\Pi^{\theta})} = \left(\frac{1-\rho}{(1-\alpha\Pi^{\theta}\rho)^2}\right)\frac{\sum_j \xi_j \hat{z}_j \left(1 + \hat{z}_j \frac{1-\alpha\Pi^\theta}{1-\alpha\Pi^\theta\rho}\right)^{-\theta}}{\sum_j \xi_j \left(1 + \hat{z}_j \frac{1-\alpha\Pi^\theta}{1-\alpha\Pi^\theta\rho}\right)^{1-\theta}}. \tag{12}$$

4. $\text{sgn}\{\Pi^\star\} = \text{sgn}\{\sum_j \xi_j \hat{z}_j \left(1 + \hat{z}_j \frac{1-\alpha\Pi^\theta}{1-\alpha\Pi^\theta\rho}\right)^{-\theta}\} = -1$ □

▸ slides

References

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Steiner, J., C. Stewart, and F. Matejka (2017). Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica 85*(2), 521–553.