Control costs, rational inattention and retail price dynamics

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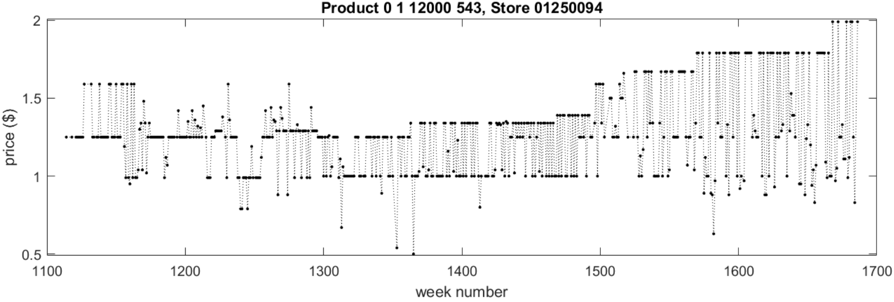
Views expressed here are personal and do not necessarily coincide with our employer's views.

A paradox: “stickiness” but “jumpiness” of retail prices



- A typical supermarket price trajectory is:
  - **Sticky**: it often revisits the exact same nominal values (“price points”)
  - **Volatile**: it frequently makes big jumps between price points
- What causes stickiness+volatility? Does it matter for business cycles?

Existing theories of retail pricing with sales

- Several theories of **sticky plans** with multiple price points
  - **Multiple menu costs**: larger menu cost to change the plan, a smaller one to jump between price points (Eichenbaum et al, '11; Álvarez/Lippi '19)
  - **Stochastic price discrimination**: heterogeneous price elasticities (Varian '80; Guimaraes/Sheedy '11; Kehoe and Midrigan, '15)
  - **"Rational inattention"**: information processing costs cause randomization across discrete price points within a plan (Matejka '16, Stevens '19)
- **Issue:** we don't find much evidence of changes between **plans**.
  Instead: sticky price points that are updated one at a time.

Goal: model sticky+volatile prices, macro implications?

- Present evidence on **sticky price points** vs. **sticky plans**
- Extend our **"control cost" (CC)** model of sticky prices (JMCB, 2019) to generate multiple sticky price points
- Extend **results linking RI with CC** (Steiner/Stewart/Matejka 2017)
  - Define a computable **limited memory RI** framework which **approximates RI**
  - Show that the model generates sticky price points
- Assess importance of price discrimination vs. costly information
- Later: assess importance of this type of pricing for monetary transmission

Some related literature

- State-dependent prices meet microdata
  - Golosov/Lucas '07, Klenow/Malin '10, Midrigan '11, Costain/Nakov '11/'19, Alvarez/Lippi/Paciello '12, Nakamura/Steinsson '15, Cavallo '18
- Stochastic price discrimination and sales
  - Varian '80, Guimaraes/Sheedy '11
- Multiple menu costs and sales
  - Eichenbaum et al '11, Kehoe/Midrigan '15, Alvarez/Lippi '19
- Discrete logit solutions to rational inattention models
  - Matejka/McKay '15, Steiner/Stewart/Matejka '17
- Rational inattention and sticky prices
  - **Matejka '16** (discrete prices), **Stevens '19** (sticky plans)

Two closely related papers

- Stevens '19: sticky plans
  - In her model all prices on the plan change simultaneously; in the data new price points are introduced one at a time
  - Her model is not standard RI – it has two different costs: a fixed cost to obtain full information and replan, plus the usual flow information cost
- Matejka '16: his choice set is the real line
  - Does not specify whether the model is nominal or real
  - If real, then he obtains real stickiness, not nominal
  - If nominal, then nominal payoffs must be stationary to get nominal stickiness
  - Since payoffs are non-stationary in the data, Matejka's model does not apply
  - Our approach: leaving the nominal price unchanged is one possible action

Outline

1. **Microdata**
   - Hard to find evidence of shifts across plans; instead sticky price points
2. **Theory**
   - CC and RI both give logit solutions
   - Equivalence: RI=CC+optimal default distribution: simplifies solution of RI
   - RI implies discretization of the choice set into price points
3. **Model of sticky price points**
   - Limited memory version of RI is computable (and gives an extra parameter)
   - We define "no change" as a possible action: stationary choice set
4. **Numerical findings**
   - **Stickiness** of price points caused by **costs of information processing**
   - **Volatility** across price points driven mainly by **customer heterogeneity**

1. Microdata on Price Trajectories

A closer look at price trajectories

- At time t, define **backwards and forwards windows** for store i, product j:
  - $\mathcal{B}_T^{i,j,t-1} \equiv$ set of all prices observed from $t - T$ to $t - 1$
  - $\mathcal{F}_T^{i,j,t+1} \equiv$ set of all prices observed from $t + 1$ to $t + T$

- **Classify price changes** $(P_{i,j,t} \neq P_{i,j,t-1})$:
  - **Transitory** if $P_{i,j,t} \notin \mathcal{B}_T^{i,j,t-1}$ and $P_{i,j,t} \notin \mathcal{F}_T^{i,j,t+1}$
  - **Recurrence** if $P_{i,j,t} \in \mathcal{B}_T^{i,j,t-1}$
  - **Introduction** if $P_{i,j,t} \notin \mathcal{B}_T^{i,j,t-1}$ and $P_{i,j,t} \in \mathcal{F}_T^{i,j,t+1}$

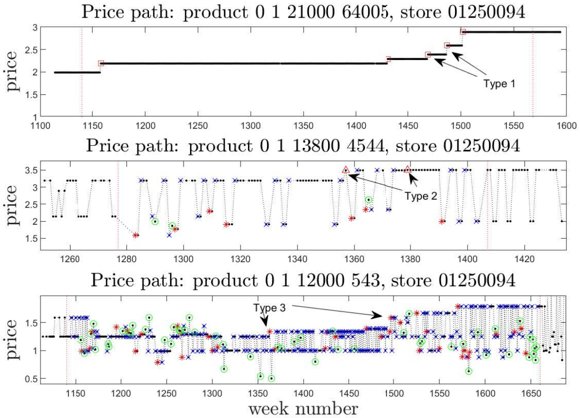
- Further **classify price introductions**:
  - **Type 1:** $\mathcal{B}_T^{i,j,t-1} \cap \mathcal{F}_T^{i,j,t+1} = \emptyset$, and **no recurrences** in $\mathcal{B}_T^{i,j,t-1}$ nor $\mathcal{F}_T^{i,j,t+1}$
    - Typical of *single price policies*
  - **Type 2:** $\mathcal{B}_T^{i,j,t-1} \cap \mathcal{F}_T^{i,j,t+1} = \emptyset$, and **recurrences occur** in $\mathcal{B}_T^{i,j,t-1}$ or $\mathcal{F}_T^{i,j,t+1}$
    - Typical of *sticky plans*
  - **Type 3:** $\mathcal{B}_T^{i,j,t-1} \cap \mathcal{F}_T^{i,j,t+1} \neq \emptyset$.
    - Typical of *sticky price points*

Products with frequent type 1/2/3 introductions: US IRi

Type 1 intros:

Type 2 intros:

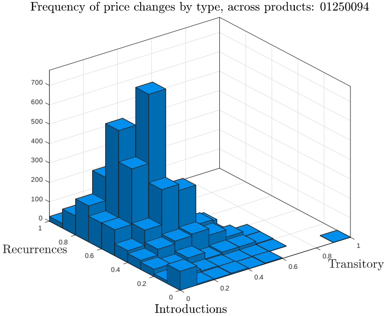
Type 3 intros:

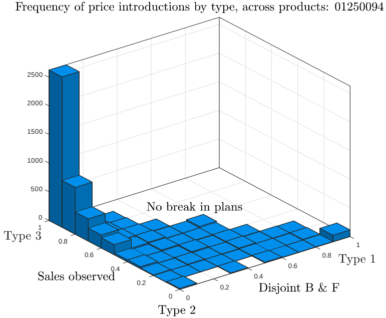


Classifying price changes:		*Classifying introductions:*	
Transitory:	green	Type 1:	squares
Recurrences:	blue	Type 2:	triangles
Introductions:	red	Type 3:	stars

Frequencies of different price change events

Figure: Histograms of event frequencies, across products (example: store 01250094).





Each point on the simplex shows frequency of transitory/recurrences/introductions for a given product. Height of bars represents number of products in each bin.

→ **Many price changes are recurrences.**

Each point on the simplex shows frequency of type 1/2/3 introductions for a given product. Height of bars represents number of products in each bin.

→ **Most introductions are Type 3.**

2. Theory: Costly Decision-Making

Control costs: precise decisions are costly

- **Making a decision** means **allocating probability** $\pi(a)$ over the set of **feasible actions** $a \in \mathcal{A}$.

- **Control cost (CC)** models are **full information** decisions where **precision is costly**:

$$V(\theta) = \max_{\pi \in \Delta(\mathcal{A})} E_\pi u(a, \theta) - \kappa \mathcal{D}(\pi || \eta). \quad (1)$$

- Here we assume **precision** is measured by **relative entropy**:

$$\mathcal{D}(\pi || \eta) \equiv \sum_{a \in \mathcal{A}} \pi(a) \ln \left(\frac{\pi(a)}{\eta(a)} \right).$$

- Calibration requires an **exogenous "benchmark"** action distribution $\eta(a)$ that applies if no decision costs are paid.

Control costs imply weighted logits

- **Expand out** the CC problem:

$$V(\theta) \quad = \quad \max_{\pi(a)} \sum_{a\in\mathcal{A}} \pi(a)u(a,\theta) - \kappa \sum_{a\in\mathcal{A}} \pi(a)\ln\left(\frac{\pi(a)}{\eta(a)}\right)$$
$$\text{s.t.} \quad \sum_{a\in\mathcal{A}} \pi(a) = 1 \quad \text{and} \quad \pi(a) \geq 0, \;\; \forall a \in \mathcal{A}. \tag{2}$$

- FOC for $\pi(a)$:

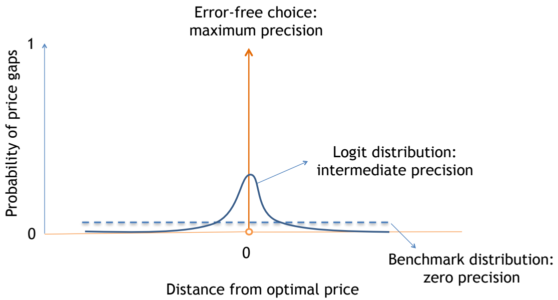
$$u(a,\theta) - \kappa\left(1 + \ln\left(\frac{\pi(a)}{\eta(a)}\right)\right) - \mu \; = \; 0,$$

- Yields a **weighted multinomial logit** (Mattsson/Weibull '02):

$$\pi(a|\theta) \; = \; \frac{\eta(a)\exp(\kappa^{-1}u(a,\theta))}{\sum_{a'\in\mathcal{A}}\eta(a')\exp(\kappa^{-1}u(a',\theta))}\,. \tag{3}$$

  - Here u may be a static payoff, or it may be the **continuation value** of a **dynamic problem**

Nested logits: sequencing of choices is irrelevant



Proposition 1.

Sequence of decisions is irrelevant in dynamic problem:

- *Choose probability of adjustment, then choose distribution over new prices*
- *Choose distribution over new prices, then choose probability to adjust*
- *Choose simultaneously*

Link to Prop. 1 details

Rational inattention: information is costly

- **Rational inattention (RI)** models interpret decision costs in a more specific way, as a **cost of information**:

$$U(\pi_\theta(\theta)) = \max_{\pi(a|\theta)\in\Delta(\mathcal{A})} E_{\pi(a,\theta)} u(a,\theta) - \kappa\mathcal{I}(a;\theta) \tag{4}$$

- Here costs depend on **mutual information** $I(a,\theta)$ between the action a and the state θ:

$$\mathcal{I}(a,\theta) \equiv \mathcal{D}(p(a,\theta)||p_a(a)p_\theta(\theta)) = \sum_{a\in\mathcal{A}}\sum_{\theta\in\Theta}\pi(a,\theta)\ln\left(\frac{\pi(a,\theta)}{\pi_a(a)\pi_\theta(\theta)}\right)$$

  - Mutual information is a **special case** of relative entropy
  - Mutual information measures **deviation from independence**

RI endogenizes the weights in the logit

- Rewrite the **RI problem** in expanded form:

$$U(\pi_\theta(\theta)) = \max_{\pi(a|\theta)\in\Delta(\mathcal{A})} \sum_{\theta\in\Theta} \pi_\theta(\theta) \sum_{a\in\mathcal{A}} \pi(a|\theta) \left[u(a,\theta) - \kappa \ln\left(\frac{\pi(a|\theta)}{\pi_a(a)}\right)\right]$$

$$\text{s.t. } \pi_a(a) = \sum_{\theta\in\Theta} \pi(a|\theta)\pi_\theta(\theta). \tag{5}$$

- Notice:
  - the **RI problem** is just an expectation across various **CC problems!**
  - $\eta(a) = \pi_a(a)$: optimal "benchmark" is the **unconditional action distribution**

- Therefore the solution is:

$$\pi(a|\theta) = \frac{\pi_a(a)\exp(\kappa^{-1}u(a,\theta))}{\sum_{a'\in\mathcal{A}} \pi_a(a')\exp(\kappa^{-1}u(a',\theta))}, \tag{6}$$

$$\pi_a(a) = \sum_{\theta\in\Theta} \pi(a|\theta)\pi_\theta(\theta). \tag{7}$$

Equivalence between RI and CC: static case

Proposition 2.

(Matejka and McKay, 2015). **RI problem (4)-(5)** *represents an expectation across* **CC problems** *under an optimally-chosen benchmark distribution:*

$$U(\pi_\theta(\theta)) = \max_{q(a)\in\Delta(\mathcal{A})} \sum_{\theta\in\Theta} \pi_\theta(\theta) \max_{\pi(a|\theta)\in\Delta(\mathcal{A})} \sum_{a\in\mathcal{A}} \pi(a|\theta) \left[u(a,\theta) - \kappa \ln\left(\frac{\pi(a|\theta)}{q(a)}\right)\right] \tag{8}$$

Therefore, the RI problem is solved by a **weighted multinomial logit**, *with weights equal to the* **marginal probabilities** *of each action:*

$$\pi(a|\theta) = \frac{q(a)\exp(\kappa^{-1}u(a,\theta))}{\sum_{a'\in\mathcal{A}} q(a')\exp(\kappa^{-1}u(a',\theta))}, \tag{9}$$

$$q(a) = \sum_{\theta\in\Theta} \pi(a|\theta)\pi_\theta(\theta). \tag{10}$$

- (9): conditional action probabilities are weighted logits
- (10): optimal weights are unconditional action frequencies

Equivalence between RI and CC: Full and Limited Memory

Proposition 3.

(Steiner, Stewart, and Matejka, 2017). A **dynamic RI problem** *with memory of all previous actions* a^{t-1} *represents an expectation across* **dynamic CC problems** *under an optimally-chosen benchmark distribution* $q(a_t|a^{t-1})$ *at each possible information set* a^{t-1}.

▸ Link to proposition details

Proposition 4.

Short-term memory rational inattention (STMRI)*: A* **dynamic RI problem** *with memory of actions* $\mathcal{B}_\tau^{t-1}$ *in the last* τ *periods only represents an expectation across* **dynamic CC problems** *under an optimally-chosen benchmark distribution* $q(a_t|\mathcal{B}_\tau^{t-1})$ *at each limited information set* $\mathcal{B}_\tau^{t-1}$.

▸ Link to proposition details

Inferring logit weights from data

- Weights on action $a \in \mathcal{A}$ represent average frequencies conditional on the information set a^{t-1}:

$$q(a|a^{t-1}) = \sum_{\theta^t} \pi(a|\theta^t, a^{t-1})\pi(\theta^t|a^{t-1})$$

  - **Not feasible** to infer $q(a|a^{t-1})$ from the data, because we can't repeatedly observe each possible information set a^{t-1}.
  - Exact RI model is hard to identify and hard to compute

- But it is simple to **infer the weights on average**:

$$\bar{q}(a) = \sum_{\theta^t} \sum_{a^{t-1}} \pi(a|\theta^t, a^{t-1})\pi(\theta^t, a^{t-1}) \approx \text{sample frequency of } a$$

  - **Approximate RI model based on average weights** is easy to identify and is feasible to compute

To infer weights, define a stationary action space

- Modelling the action space in a stationary way is crucial
- **Example**: Event $P_{i,j,t} =$ €3.69 may be rare in the dataset, and may vary a lot across product, store, time period, and history of actions.
  - Hard to infer $\pi(P_{i,j,t} =$ €3.69) directly from the data.
- **Example**: Event $P_{i,j,t} \neq P_{i,j,t-1}$ is common in the dataset, and may not vary much with product, store, time period, and history of actions.
  - Might infer $\pi(P_{i,j,t} \neq P_{i,j,t-1})$ directly from the data.

Discreteness of RI solution

- Necessary conditions for RI action distribution:

$$\pi(a|\theta) = \frac{q(a)\exp(\kappa^{-1}u(a,\theta))}{\sum_{a'\in\mathcal{A}} q(a')\exp(\kappa^{-1}u(a',\theta))},$$
$$q(a) = \sum_{\theta\in\Theta} \pi_\theta(\theta)\pi(a|\theta).$$

- But **not all actions** a are chosen with positive probability.
  - Some actions a may have $q(a) = \pi(a|\theta) = 0$ for all θ.

Proposition 5.

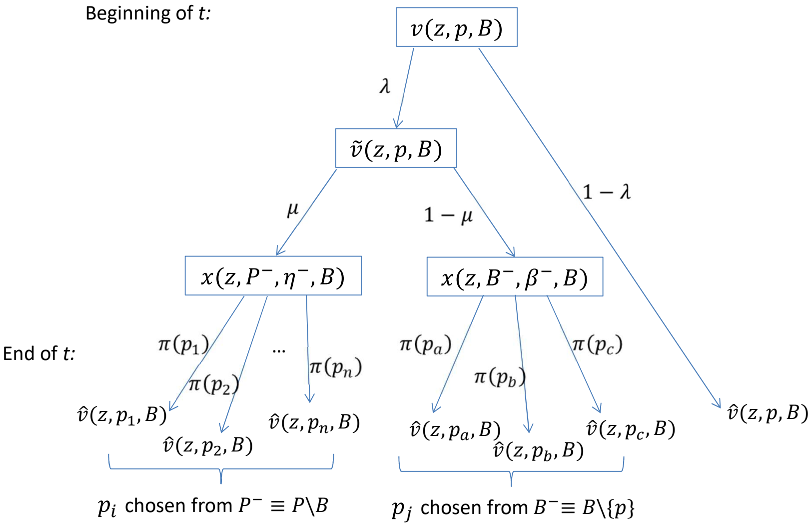
(Fix, 1978). The benchmark distribution $q(a) \geq 0$ solves (5) if it satisfies

$$F(a) \equiv \frac{\sum_{\theta\in\Theta}\pi_\theta(\theta)\sum_{a\in\mathcal{A}}\exp(\kappa^{-1}u(a,\theta))}{\sum_{a'\in\mathcal{A}} q(a')\exp(\kappa^{-1}u(a',\theta))} \leq 1, \quad (11)$$

at all $a \in \mathcal{A}$, with equality wherever $q(a) > 0$ strictly.

- Must check (11) to see which actions have nonzero probability.

CC model with memory: multiple sticky price points



CC model with memory: multiple sticky price points

- Let $\mathcal{P}$ be a large finite set of *real* prices, and let η be a distribution over $\mathcal{P}$.
- Let $\mathcal{B}_t^T$ be the *real* values of the T **most recent nominal prices**

- Value v of **option to adjust**:

$$v(p,z,\mathcal{B}^T) = \max_{\lambda\in[0,1]} \lambda\tilde{v}(p,z,\mathcal{B}^T) + (1-\lambda)\hat{v}(p,z,\mathcal{B}^T) - \kappa w \mathcal{D}(\lambda||\bar{\lambda}) \quad (12)$$

- Value $\tilde{v}$ of option to **choose from $\mathcal{B}^T$ or not**:

$$\tilde{v}(p,z,\mathcal{B}^T) = \max_{\mu\in[0,1]} \mu x(z,\mathcal{P}^-,\eta^-) + (1-\mu)x(z,\mathcal{B}^-,\beta^-) - \kappa w \mathcal{D}(\mu||\bar{\mu}) \quad (13)$$

$$\text{where}: \quad \mathcal{P}^- = \mathcal{P}\diagdown\mathcal{B}^T, \text{ and } \quad \eta^-(\tilde{p}) = \frac{\eta(\tilde{p})}{1-\eta(\mathcal{B}^T)}, \quad \forall\tilde{p}\in\mathcal{P}^- \quad (14)$$

$$\mathcal{B}^- = \mathcal{B}^T\diagdown\{p\}, \text{ and } \quad \beta^-(\tilde{p}) = \frac{\#(\tilde{p}\in\mathcal{B}^T)}{T-\#(p\in\mathcal{B}^T)}, \quad \forall\tilde{p}\in\mathcal{B}^- \quad (15)$$

Link to neural nets algorithm

CC model with memory: multiple sticky price points

- Value x of **setting a new price**:

$$x(z, \mathcal{X}, \xi) \ = \max_{\pi \in \Delta(\mathcal{X})} \sum_{\tilde{p} \in \tilde{\mathcal{X}}} \pi(\tilde{p}) \hat{v}(\tilde{p}, z, \mathcal{B}^T) - \kappa w \mathcal{D}(\pi || \xi) \tag{16}$$

- Value $\hat{v}$ of **producing at price** p:

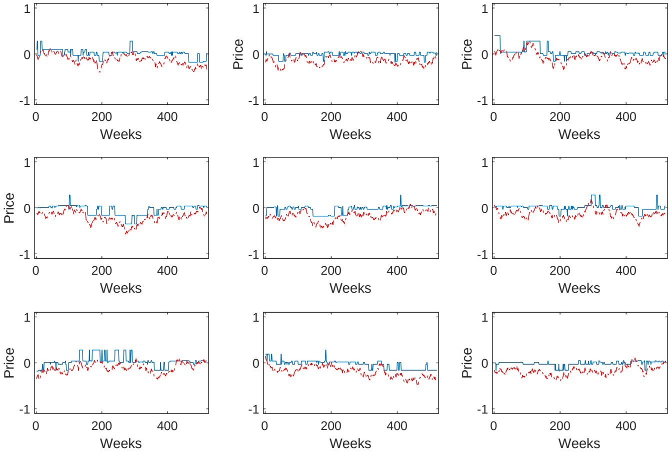
$$\hat{v}(p, z, \mathcal{B}^T) \ = u(z, p) + \frac{1}{1+r} \sum_{z'} \pi_z(z'|z) v(p - i, z', \mathcal{B}^{T\prime}) \tag{17}$$

- **Updating price points** $\mathcal{B}^T \equiv \{b_1, b_2, \ldots b_T\}$ as follows:

$$\mathcal{B}^T \equiv \{p - i,\ b_1 - i,\ b_2 - i, \ldots b_{T-1} - i\}$$

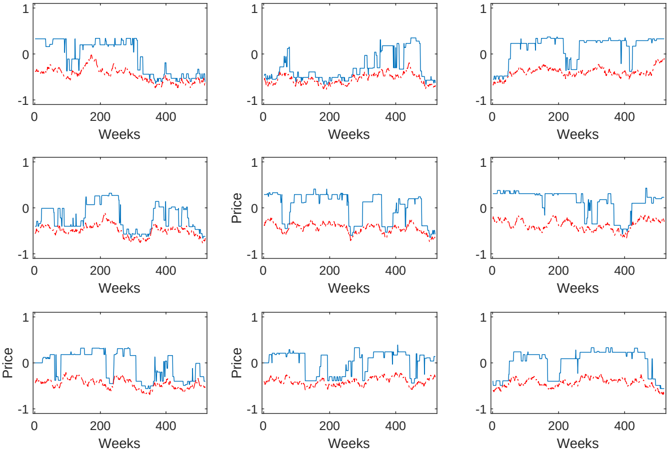
Link to neural nets algorithm

Simulated price trajectories: $\epsilon = 7$ with memory



- Lots of stickiness
- Sometimes previous price points are repeated

Simulated price trajectories: $\epsilon \in \{3, 11\}$ with memory



- Frequent repetition of previous price points
- Markups rise when marginal cost rises

Some summary statistics and conclusions

Table: Model specifications and simulation results (medians)

Parameterizations:	$\epsilon = 7$ No mem.	$\epsilon = 7$ Mem.	$\epsilon = 3, 11$ No mem.	$\epsilon = 3, 11$ Mem.
Price changes				
Adj. frequency (weekly)	0.155	0.138	0.157	0.152
Classifying price changes				
Frequency of recurrences	33.3	67.2	40.0	66.2
Freq. of type 3 introductions	75.0	90.9	75.7	93.6
Short-run volatility ratios				
Ratio VR^{avg}	0.98	0.95	0.85	1.33
Ratio VR^{diff}	16.5	17.5	14.1	23.3
Ratio VR^{reg}	1.52	1.68	1.36	2.36
Ratio VR^{abs}	6.8	7.2	5.6	10.4

- Decision costs with **finite memory** helps explain **stickiness** (frequent recurrences and frequent Type 3 introductions)
- **Heterogeneous demand** (elasticity 3 vs. 11) helps explain excess **volatility**

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Conclusions

- **Explaining nominal stickiness:**
  - **Decision costs** help explain intermittent nominal price adjustment
  - **Finite memory** helps explain recurrence of previous nominal prices
  - **Price discrimination** needed to explain excess short-run volatility
- **Computing models of costly decisions**
  - In general, rational inattention models are hard to compute
  - We defined **short-term memory rational inattention (STMRI)**
  - Showed how STMRI can be computed using neural networks
  - Defining a **realistic, stationary action set** makes it possible to identify model parameters
- **RI can only be tested jointly** with a hypothesis about the nature of the action set
  - With an arbitrary action set, RI does not generate nominal stickiness
  - **STMRI, on a realistic action set, can explain nominal stickiness,** including retail sales behavior

Thanks for your (costly) attention!

APPENDIX: Detailed propositions

Invariance to decision sequence

- A convenient fact about the **relative entropy cost function** is that **decisions are invariant to changes in sequencing**, if benchmark distributions are appropriately defined.

Proposition 1.

Let $V(\theta; \mathcal{A}, \eta)$ be the value of (1) with choice set $\mathcal{A}$ and benchmark distribution η.

Split up the action set $\mathcal{A}$ into a **partition** $\mathcal{A}^0 \equiv \{\mathcal{B}_1, \mathcal{B}_2 ..., \mathcal{B}_n\}$.
Define $\eta^0(\mathcal{B}_i) \equiv \sum_{a \in \mathcal{B}_i} \eta(a)$, and $\xi_i(a) \equiv \eta(a)/\eta^0(\mathcal{B}_i)$ for each $a \in \mathcal{B}_i$.

Consider the **two-step CC problem***:*

$$V^0(\theta; \mathcal{A}^0, \eta^0) \quad = \quad \max_{\pi^0 \in \Delta(\mathcal{A}^0)} \; E_{\pi^0} V(\theta; \mathcal{B}, \xi) - \kappa \mathcal{D}(\pi^0 || \eta^0). \tag{18}$$

Two-step problem (18) has the same solution as the one-step problem (1).

▸ Back to main presentation

Equivalence between RI and CC: dynamic case

Proposition 3.

(Steiner, Stewart, and Matejka, 2017). Consider a dynamic RI problem:

$$U(a^0) = \max_{\pi(a_t|\theta^t, a^{t-1}) \in \Delta(\mathcal{A})} E\left[\sum_{t=1}^{\infty} \delta^t \left(u(a_t, \theta_t) - \kappa \mathcal{I}(a_t, \theta^t | a^{t-1}) \right) \middle| a^0 \right]. \tag{19}$$

(i.) *Problem (19) is equivalent to the following double optimization:*

$$U(a^0) = \max_{\pi, q} E\left[\sum_{t=1}^{\infty} \delta^t \left(u(a^t, \theta^t) - \kappa \log\left(\frac{\pi(a_t|\theta^t, a^{t-1})}{q(a_t|a^{t-1})} \right) \right) \middle| a^0 \right]. \tag{20}$$

(ii.) *Problem (20) represents an expectation across* **full-info CC problems** *under an optimal benchmark distribution q:*

$$U(a^{t-1}) \;=\; \delta \max_{q(a|a^{t-1}) \in \Delta(\mathcal{A})} \sum_{\theta^t} \pi(\theta^t | a^{t-1}) V(\theta^t; a^{t-1}, q). \tag{21}$$

(Continues)

Equivalence between RI and CC: dynamic case (continued)

Proposition 3.

(iii.) *In (21), $V(\theta^t; a^{t-1}, q)$ is the value of a recursive, full-info CC problem:*

$$V(\theta^t; a^{t-1}, q) = \max_{\pi(a_t|\theta^t, a^{t-1}) \in \Delta(\mathcal{A})} \sum_{a_t \in \mathcal{A}} \pi(a_t|\theta^t, a^{t-1}) \Bigg[u(a_t, \theta_t) \dots$$

$$-\kappa \ln \left(\frac{\pi(a_t|\theta^t, a^{t-1})}{q(a_t|a^{t-1})} \right) + \delta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) V(\theta^{t+1}; a^t, q) \Bigg]. \tag{22}$$

(iv.) *Hence, (19) and (22) are solved by a* **weighted multinomial logit***:*

$$\pi(a_t|\theta^t, a^{t-1}) \; = \; \frac{q(a_t|a^{t-1}) \exp(\kappa^{-1} \hat{v}(a_t, \theta^t; a^{t-1}, q))}{\sum_{a' \in \mathcal{A}} q(a'|a^{t-1}) \exp(\kappa^{-1} \hat{v}(a', \theta^t; a^{t-1}, q))}, \tag{23}$$

where

$$\hat{v}(a_t, \theta^t; a^{t-1}, q) \; \equiv \; u(a_t, \theta_t) + \delta \sum_{\theta'} \pi(\theta'|\theta^t) V(\theta'; a^t, q). \tag{24}$$

(v.) *The optimal q is the marginal distribution, conditional on signals observed:*

$$q(a_t|a^{t-1}) \; = \; \sum_{\theta^t} \pi(a_t|\theta^t, a^{t-1}) \pi(\theta^t|a^{t-1}). \tag{25}$$

Back to main presentation

Equivalence between STMRI and CC

Proposition 4.

Consider a **short-term memory rational inattention (STMRI)** *problem:*

$$U(\mathcal{B}_\tau^0) = \max_{\pi(a_t|\theta^t, \mathcal{B}_\tau^{t-1}) \in \Delta(\mathcal{A})} E\left[\sum_{t=1}^{\infty} \delta^t \left(u(a_t, \theta_t) - \kappa \mathcal{I}(a_t, \theta^t | \mathcal{B}_\tau^{t-1})\right) \middle| \mathcal{B}_\tau^0\right]. \quad (26)$$

(a.) *The results of Prop. 3 concerning the unlimited memory RI problem (19) extend to the STMRI problem (26). In particular, (26) is solved by a weighted multinomial logit:*

$$\pi(a_t|\theta^t, \mathcal{B}_\tau^{t-1}) = \frac{q(a_t|\mathcal{B}_\tau^{t-1}) \exp(\kappa^{-1} \hat{v}(a_t, \theta^t; \mathcal{B}_\tau^{t-1}))}{\sum_{a' \in \mathcal{A}} q(a'|\mathcal{B}_\tau^{t-1}) \exp(\kappa^{-1} \hat{v}(a', \theta^t; \mathcal{B}_\tau^{t-1}))}, \quad (27)$$

where

$$q(a_t|\mathcal{B}_\tau^{t-1}) = \sum_{\theta^t} \pi(a_t|\theta^t, \mathcal{B}_\tau^{t-1}) \pi(\theta^t|\mathcal{B}_\tau^{t-1}), \quad (28)$$

and $\hat{v}$ is derived from the value function of a dynamic CC problem, as in (24).

(b.) *As memory increases ($\tau \to \infty$), the probabilities and value functions that solve the STMRI problem (26) converge to the solution of the unlimited memory RI problem (19).*

Back to main presentation

How to calibrate the benchmark parameters?

1. **Exogenous uniform benchmark** (Costain/Nakov '19):
   - Benchmark hazard $\bar{\lambda}$ is a constant to be estimated
   - $\mathcal{P}$ is a uniform grid of log real prices
   - The benchmark distribution η is uniform on $\mathcal{P}$

2. **Empirical RI-CC hybrid:**
   In this paper, we instead set the **benchmarks equal to their sample averages** in the data.
   Consider a subset of products I that appear to follow *single price policies*, that is, they only display *Type-1* price introductions.
   - $\bar{\lambda}_t = \bar{\lambda} =$ sample average adjustment frequency in set I.
   - $\tilde{\eta}_t(p) = \tilde{\eta}(p) =$ average histogram, across products in set I, of newly-set log prices, as deviations from product-specific mean.

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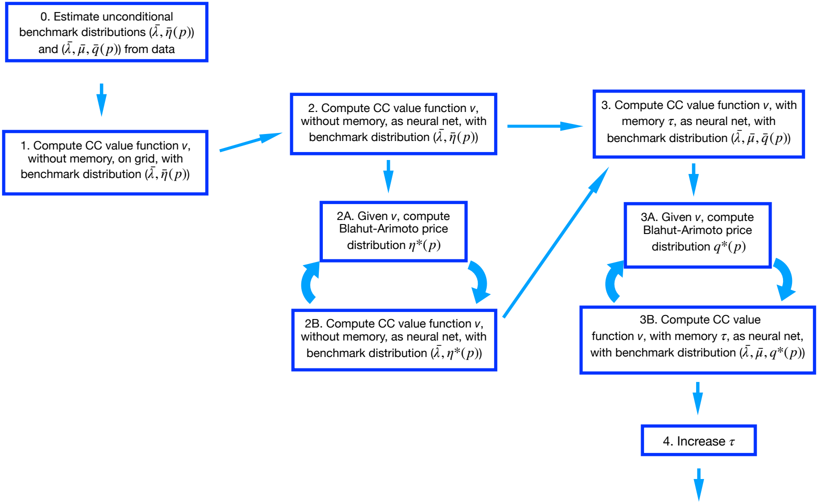
- To calculate $\tilde{\eta}(p)$:
  1. Find the vector of all new nominal prices $P_{i,j,t}$ chosen for a given product i ;
  2. Calculate the product-specific mean $\bar{P}_i$;
  3. Demean the prices to obtain $p_{i,j,t} = \log(P_{i,j,t}/P_t)$ at all times t such that the price of product i changed at store j;
  4. Aggregate the histograms of prices $p_{i,j,t}$ across all products $i \in I$.

Back to main presentation

Neural nets algorithm: various options



Back to main presentation