Strike while the Iron is Hot:

Optimal Monetary Policy with a Nonlinear Phillips Curve

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Motivation

- The recent inflation surge featured
  - Increase in the frequency of price changes (Montag and Villar, 2023) US
  - Increase in Phillips curve slope (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023) US
- Optimal monetary policy is mainly studied in models, in which the Phillips curve is linear and the frequency is held constant (Galí, 2008; Woodford, 2003)
- What does optimal monetary policy look like with a nonlinear Phillips curve and endogenous variation in frequency? How should CBs respond to a large inflation surge?

What do we do?

- We use the standard state-dependent pricing model of Golosov and Lucas (2007)
- Solve it nonlinearly using a new algorithm over the sequence space
- Positive analysis under a Taylor rule
  - Trace the responses to shocks of different sizes
  - Assess the nonlinearity of the Phillips curve
- Normative analysis: Ramsey optimal policy
  - Optimal long-run inflation
  - Trace the optimal responses to shocks
  - Characterize the (nonlinear) targeting rule after *large* cost-push shocks

What do we find?

- In this model the Phillips curve is nonlinear: it gets steeper as frequency increases
- In response to small shocks, optimal monetary policy is similar to the one under Calvo
- In response to efficiency shocks, there is divine coincidence, as in Calvo
- Different responses to small and large cost-push shocks. Optimal policy leans aggressively against inflation when frequency rises: "it strikes while the iron is hot"

Literature

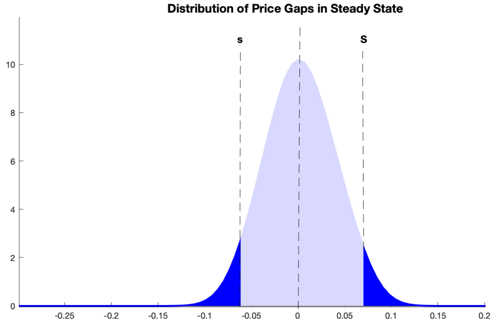
- Nonlinear Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023)
  - Microfounded by state-dependent price setting (Golosov and Lucas, 2007; Gertler and Leahy, 2008; Auclert et al., 2022)
  - In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020; Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)
- Optimal policy in a menu cost economy
  - Optimal inflation target (Burstein and Hellwig, 2008; Adam and Weber, 2019; Blanco, 2021)
  - Small shocks, large shocks, optimal nonlinear target rule (comp. Galí, 2008; Woodford, 2003)
  - Focus on aggregate shocks (unlike Caratelli and Halperin, 2023, who study *sectoral* shocks)

Overview of the model

- Heterogeneous-firm DSGE model with fixed costs of price-adjustment
- Households: consume a Dixit-Stiglitz basket of goods, and work HH
- Firms: produce differentiated goods using labor only and are subject to aggregate TFP shocks and idiosyncratic "quality" shocks. They have market power and set prices optimally subject to a fixed cost (Golosov and Lucas, 2007) Firms.
- Monetary policy: either follows Taylor rule or set optimally to maximize household welfare under commitment Policy

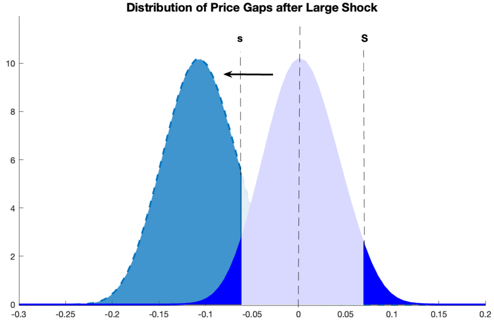
Model: Intuitive summary

- Each period, firms choose whether to reset their price and, if so, what new price to set
- The firm's optimality conditions define the reset price and the inaction region (s,S)
- Given the idiosyncratic shock, this endogenously determines the price distribution
- Let $p_t(j) \equiv \log\left(P_t(j)/(A_t(j)P_t)\right)$ be the quality-adjusted log relative price
- Let $x_t(j) \equiv p_t(j) - p_t^*(j)$ be the price gap



Model under large shock

- Large aggregate shock: shifts the distribution of price gaps for all firms
- Limited impact on the (s,S) bands
- Pushes a large fraction of firms outside of the inaction region
- Large increase in the frequency of price changes and hence additional flexibility of the aggregate price level (on top of "selection")



Model: Distortions

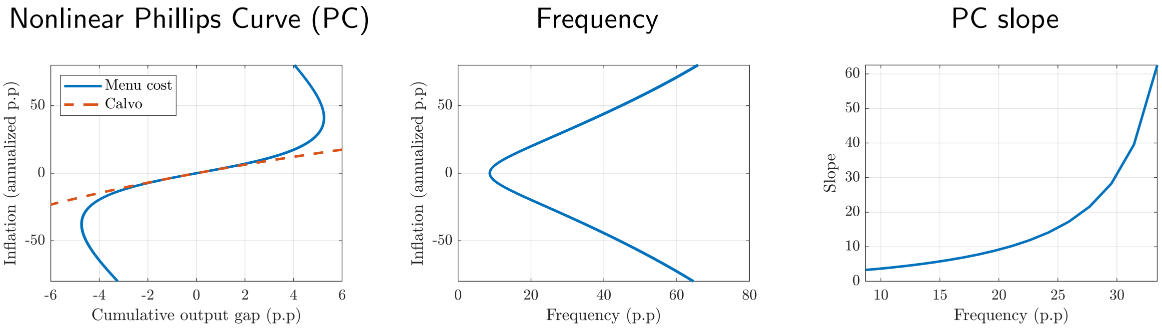
- Monopolistic competition and nominal frictions imply three distortions:
  - Inefficient markup fluctuations
  - Price dispersion
  - Price adjustment (menu) costs

Calibration

		Households	
β	$0.96^{1/12}$	Discount rate	Golosov and Lucas (2007)
ϵ	7	Elasticity of substitution	Golosov and Lucas (2007)
γ	1	Risk aversion parameter	Midrigan (2011)
υ	1	Utility weight on labor	Set to yield $w = C$
		Price setting	
η	3.6%	Menu cost	Set to match 8.7% of frequency
σ	2.4%	Std dev of quality shocks	and 8.5% size in Nakamura and Steinsson (2008)
		Monetary policy	
ϕ_π	1.5	Inflation coefficient in Taylor rule	Taylor (1993)
ϕ_y	0.5/12	Output gap coefficient in Taylor rule	Taylor (1993)
ρ_i	$0.75^{1/3}$	Smoothing coefficient	
		Shocks	
ρ_A	$0.95^{1/3}$	Persistence of the TFP shock	Smets and Wouters (2007)
ρ_τ	$0.9^{1/3}$	Persistence of the cost-push shock	Smets and Wouters (2007)

Nonlinearity of the Phillips Curve at realistic frequency (20%) US

Consider the model under a Taylor rule Robustness



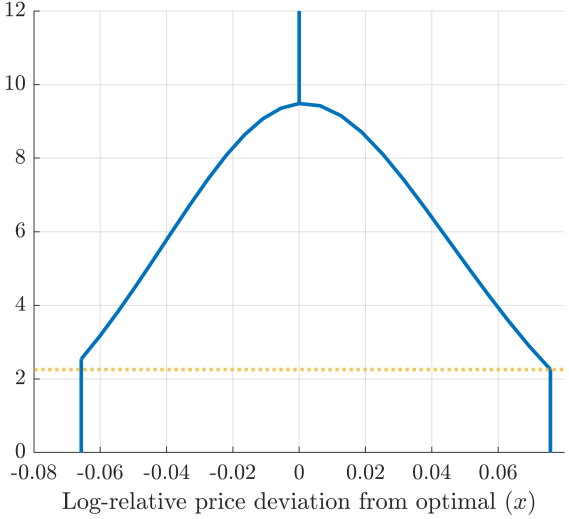
Normative results: Computation

- Challenges
  - Price change distribution and firms' value function are infinite-dimensional objects
  - In the Ramsey problem, we need derivatives w.r.t. both
- New algorithm, inspired by González et al. (2024)
  - Approximate distribution and value functions by piece-wise linear interpolation on grid
  - Endogenous grid points: (s,S) bands and the optimal reset price
  - Solve in the sequence space using Dynare (Adjemian et al. (2023))

Optimal long-run inflation rate

- The Ramsey steady-state inflation rate is slightly above zero: $\pi^* = 0.25\%$
  - Close to the inflation rate that minimizes the steady-state frequency of price changes
- Why not exactly zero as in Calvo (1983)?
  - Asymmetry of the profit function leads to asymmetric (s,S) bands: a negative price gap is less desirable than a positive price gap of the same size
  - At zero inflation, more mass around the lower (s) band than around the higher (S) band
  - Slightly positive inflation raises p^* and pushes the mass of firms to the right inside (s,S)
  - This leads to lower frequency and lower price-adjustment costs

Steady-state price distribution (at zero inflation)



Optimal response to cost-push shocks

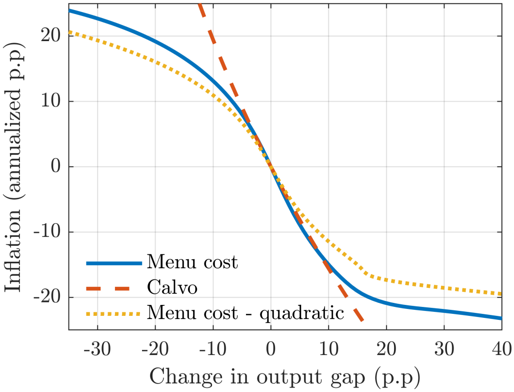
- In linearized Calvo (1983), optimal policy is a flexible inflation targeting rule

$$\pi_t = -\frac{1}{\epsilon}\Delta\tilde{y}_t$$

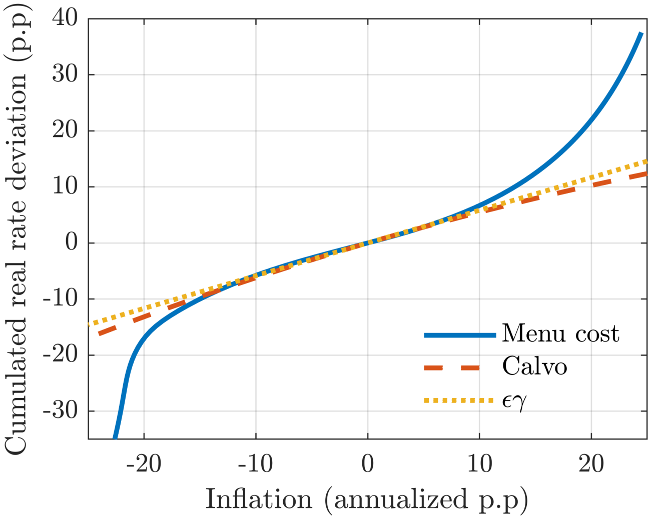
- Slope $-1/\epsilon$ is independent of the frequency of repricing or the slope of the PC
  - An increase in frequency raises the slope of the Phillips curve κ
  - But it also raises the relative weight of the *output-gap* in welfare, $\lambda = \kappa/\epsilon$
  - Why? Because more price-flexibility implies that inflation is less costly.
- For small cost-push shocks, the slope of the targeting rule in Golosov and Lucas (2007) is also $-1/\epsilon$!

Nonlinear targeting rule

- Globally, the target rule is nonlinear Robust
- After large shocks, the planner stabilizes inflation more relative to the output gap
- Why? Stabilizing inflation is cheaper due to the lower sacrifice ratio (higher freq.)
    - Similar results with quadratic objective
    - The nonlinearity of the targeting rule is due to the nonlinear Phillips curve



Nonlinear targeting rule for the real interest rate



Optimal responses to efficiency shocks: "divine coincidence"

- In the standard NK model with Calvo pricing: divine coincidence holds after TFP and other shocks affecting the efficient allocation
- Optimal policy fully stabilizes inflation and closes the output gap
- We show analytically, that, after a TFP shock, divine coincidence holds also in the menu cost model: inflation is fully stabilized at steady state and the output gap is closed

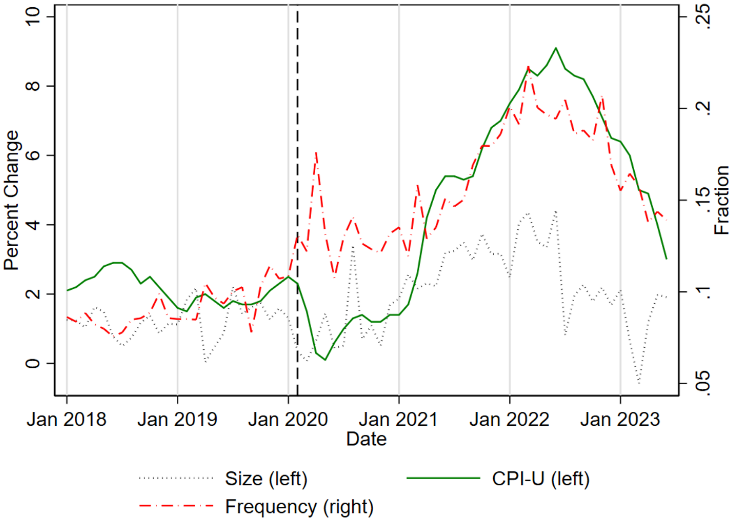
Conclusion

We study optimal policy in a state-dependent framework with a nonlinear Phillips curve

- Optimal long-run inflation is near zero (slightly positive)
- Divine coincidence holds for efficiency shocks
- For small cost shocks the optimal response is similar to Calvo (1983):

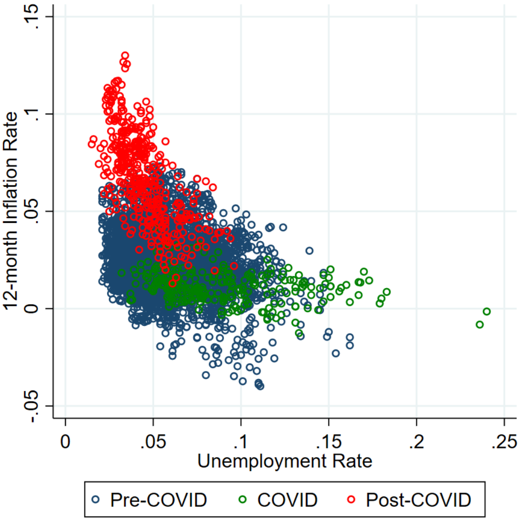
  the lower welfare weight on inflation offsets the higher slope of the Phillips curve
- For large cost shocks, CB leans aggressively against inflation: strike while the iron is hot!

CPI and frequency of price changes in the US, Montag and Villar (2023)



Back

Phillips correlation across US cities, Cerrato and Gitti (2023)



Back

Households

- A representative household consumes (C_t), supplies labor hours (N_t) and saves in one-period nominal bonds (B_t).

- The household's problem is:

$$\max_{C_t,N_t,B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C)_t - \nu N_t$$

$$\text{s.t. } P_t C_t + B_t + T_t = R_{t-1} B_{t-1} + W_t N_t + D_t,$$

where P_t is the price level, R_t is the gross nominal interest rate, W_t is the nominal wage, T_t are lump sum transfers and D_t are profits

Back

Consumption and labor

- Aggregate consumption C_t and the price level are defined as:

$$C_t = \left\{ \int \left[A_t(i) C_t(i) \right]^{\frac{\epsilon - 1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon - 1}}, \quad P_t = \left[\int_0^1 \left(\frac{P_t(i)}{A_t(i)} \right)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

where $A_t(i)$ is product quality, ϵ is the elasticity of substitution.

- Labor supply condition and Euler equation are given by:

$$W_t = \upsilon P_t C_t, \quad 1 = \mathbb{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{R_t}{\Pi_{t+1}} \right]$$

Back

Monopolistic producers

- Production of good i is given by $Y_t(i) = A_t \frac{N_t(i)}{A_t(i)}$, where quality follows a random walk

$$log(A_t(i)) = log(A_{t-1}(i)) + \varepsilon_t(i), \quad \varepsilon_t(i) \sim N(0, \sigma_t^2)$$

- Firms real profits

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)}{P_t} Y_t(i) - (1 - \tau_t) \frac{W_t}{P_t} N_t(i)$$

- Firms face a fixed cost η to update prices

Back

Quality-adjusted relative prices

- Let $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log relative price
- Real profit then is

$$\Pi(p_t(i), w_t, A_t) \equiv \frac{D_t(i)}{P_t} = C_t e^{p_t(i)(1-\epsilon)} - C_t(1-\tau_t)\frac{w_t}{A_t} e^{p_t(i)(-\epsilon)}$$

where w_t is the real wage.

- When nominal price $P_t(i)$ stays constant, $p_t(i)$ evolves: $p_t(i) = p_{t-1}(i) - \sigma_t \varepsilon_t(i) - \pi_t$

Back

Pricing decision

- Let $\lambda_t(p)$ be the price-adjustment probability
- Value function is

$$\begin{aligned} V_t(p) &= \Pi(p, w_t, A_t) \\ &+ \mathbb{E}_t\left[(1-\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}))\Lambda_{t,t+1}V_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\right] \\ &+ \mathbb{E}_t\left[\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\Lambda_{t,t+1}\left(\max_{p'} V_{t+1}(p') - \eta w_{t+1}\right)\right]. \end{aligned}$$

- The adjustment probability is

$$\lambda_t(p) = I[\max_{p'} V_t(p') - \eta w_t > V_t(p)]$$

where $I[\cdot]$ is the indicator function.

Back

Monetary Policy

- The central bank either sets policy optimally, or follows a Taylor rule:

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho_r)\left[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)\right] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2)$$

- Shocks: employment subsidy (τ_t), TFP (A_t), volatility (σ_t)

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_{A,t} \quad \varepsilon_{A,t} \sim N(0, \sigma_A^2)$$

$$\tau_t - \tau = \rho_\tau(\tau_{t-1} - \tau) + \varepsilon_{\tau,t} \quad \varepsilon_{\tau,t} \sim N(0, \sigma_\tau^2),$$

$$\log(\sigma_t/\sigma) = \rho_\sigma \log(\sigma_{t-1}/\sigma) + \varepsilon_{\sigma,t} \quad \varepsilon_{\sigma,t} \sim N(0, \sigma_\sigma^2)$$

Back

Aggregation and market clearing

- Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p)\, dp,$$

- Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p)\, dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t(p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

Back

Law of motion of the price density

$$g_t(p) = \begin{cases} (1-\lambda_t(p)) \int g_{t-1}(p+\sigma\varepsilon+\pi_t) d\xi(\varepsilon) & \text{if } p \neq p_t^*, \\ (1-\lambda_t(p_t^*)) \int g_{t-1}(p_t^*+\sigma\varepsilon+\pi_t) d\xi(\varepsilon) + \\ \quad \int_{\underline{p}}^{\overline{p}} \lambda_t(\tilde{p}) \left(\int g_{t-1}(\tilde{p}+\sigma\varepsilon+\pi_t) d\xi(\varepsilon) \right) d\tilde{p} & \text{if } p = p_t^*. \end{cases}$$

Back

The Ramsey problem

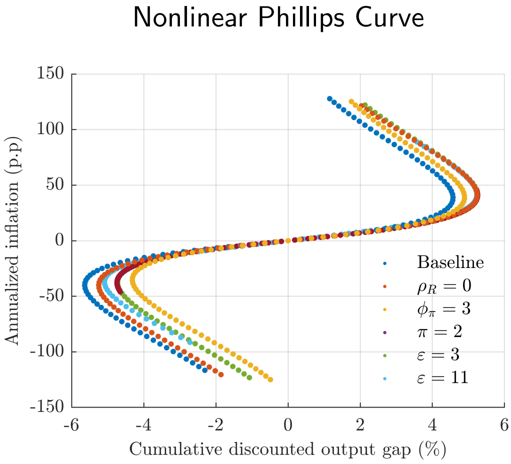
$$\max_{\{g_t^c(\cdot),g_t^0,V_t(\cdot),C_t,w_t,p_t^*,s_t,S_t,\pi_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, \frac{C_t}{A_t}\left(\int e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p)\,dx + g_t^0 e^{(p_t^*)(-\epsilon)}\right) + \eta g_t^0\right)$$

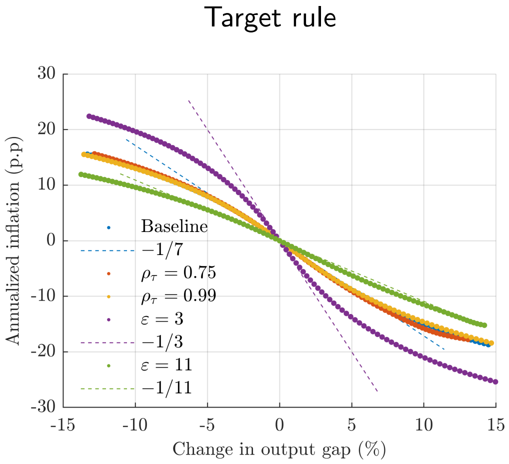
subject to

$$\begin{aligned}
1 &= \int e^{(x+p_t^*)(1-\epsilon)} g_t^c(x)\,dx + g_t^0 e^{(p_t^*)(1-\epsilon)}, \\
V_t'(0) &= \Pi_t'(x) + \frac{1}{\sigma}\Lambda_{t+1}\int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x')\frac{\partial \phi\left(\frac{x-x'-\pi_t^*}{\sigma}\right)}{\partial x}dx' + \Lambda_{t+1}\left(\phi\left(\frac{S_{t+1}-\pi_t^*}{\sigma}\right) - \phi\left(\frac{s_{t+1}-\pi_t^*}{\sigma}\right)\right)(V_{t+1}(0) - \eta w_{t+1}), \\
V_t(s_t) &= V_t(0) - \eta w_t, \\
V_t(S_t) &= V_t(0) - \eta w_t, \\
w_t &= \upsilon C_t^{\gamma}, \\
V_t(x) &= \Pi(x, p_t^*, w_t, A_t) + \Lambda_{t,t+1}\frac{1}{\sigma}\int_{s_t}^{S_t}\left[V_{t+1}(x')\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx' + \Lambda_{t,t+1}\left(1 - \frac{1}{\sigma}\int_{s_t}^{S_t}\left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx'\right)\left[(V_{t+1}(0) - \eta w_{t+1})\right], \\
g_t^c(x) &= \frac{1}{\sigma}\int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1})\phi\left(\frac{x_{-1}-x-\pi_t^*}{\sigma}\right)dx_{-1} + g_{t-1}^0\phi\left(\frac{-x-\pi_t^*}{\sigma}\right), \\
g_t^0 &= 1 - \int_{s_t}^{S_t} g_t^c(x)dx.
\end{aligned}$$

Back

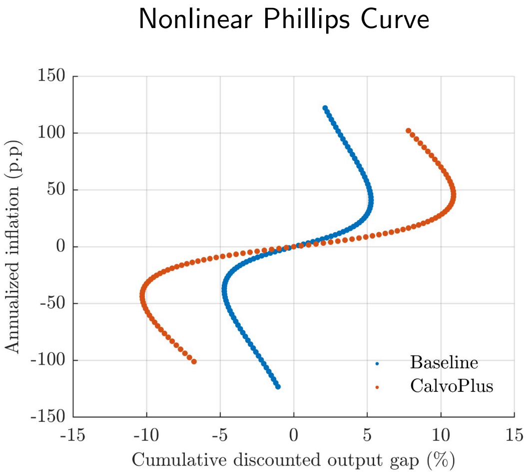
Robustness

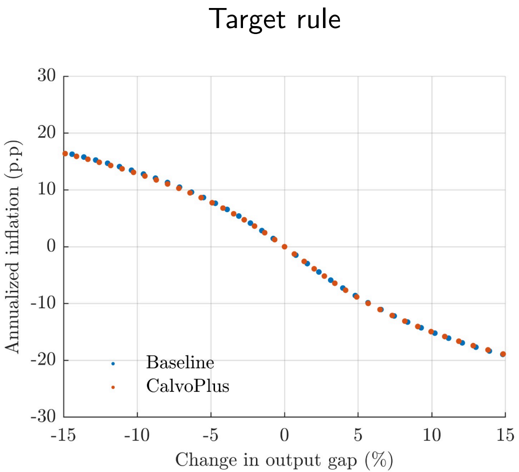




Back

CalvoPlus model





Back