Learning from Experience in the Stock Market

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Dublin, 29 July 2014

Motivation: looking for alternative to rational expectations

- Dissatisfaction with the rational expectations model: it endows people with too much knowledge about their environment.

Unrealistic (Kocherlakota, 2010): "*Using rational expectations has been attractive because it provides a simple and unified way to approach the modeling of forward-looking behavior. However, it is clearly unrealistic*"

Policymakers need a model of bubbles (Sargent, 2010): "*For policymakers to know whether and how they can moderate bubbles, we need to have well-confirmed quantitative versions of such models up and running. We are working on it...*"

Motivation: promising new approach

- Adam and Marcet (JET, 2011) introduce a weaker form of rationality: people maximize their utility given their beliefs
- "Internal rationality" does not require knowledge of the true stochastic processes for all payoff-relevant variables
- And it does not require knowledge of the complete market structure: e.g. no need to know the preferences of the marginal asset holder
- AM show that: (1) the Law of Iterated Expectations does not hold
- (2) the stock price is determined by the expected payoff one period later, and not by the discounted sum of all future dividends

Motivation: empirical models of learning

- Malmendier and Nagel (2009, 2011): evidence that people "learn from experience": they are more strongly influenced by data realized during their own lifetimes than by earlier historical data
- Individuals who experienced low stock-market returns in their lives are less likely to participate in the stock market, invest a lower fraction of their assets in stocks (MN 2011)
- Young individuals place more weight on recently experienced inflation than older individuals (MN 2009)
- Learning may be perpetual if historical data "gets lost" as new generations replace older ones

What we do

- We formalize the idea of Malmendier-Nagel in a stochastic OLG setup
- Study a simple, one-asset, Lucas tree model
- Agents learn the properties of dividends and the stock price from their own life experience – they ignore data prior to their birth
- And they weigh more heavily more recent observations

Preview of results

- Due to heterogeneity in information sets, there is **disagreement** about the future course of the stock price
- Disagreement gives rise to **trade**
- **Boom-and-bust** cycles in the P/D ratio, similar to what we see in the data ("bubbles" from the point of view of the RE model)
- Even though individuals learn with decreasing gain, the population as a whole learns approximately with constant gain
- Key for these results is the reset of learning by newborns

Related literature

- Asset pricing with learning by a representative agent: Timmermann (1993), Weitzman (2007), Cogley and Sargent (2008)
  - Agents use all available information and know ex-ante the mapping between asset prices and dividends
  - They only need to learn about dividends to achieve convergence to REE
- Convergence to REE in economies with asymmetric information: Radner (1979), Lucas (1972), Vives (1993)
  - Analyze the speed of convergence to REE
  - Whenever the average precision of private information is finite, convergence to REE is slow

Related literature

- The role of higher-order expectations for asset prices: Allen, Morris and Shin (2006)
  - In the absence of common knowledge about higher-order beliefs, asset prices typically react more sluggishly to changes in fundamentals
- Learning with heterogeneous agents: Giannitsarou (2003), Branch and McGough (2004), Branch and Evans (2006), Graham (2011)
  - In contrast individuals in our economy use the same learning scheme, have the same preferences, and observe the same public variables
  - Heterogeneity in informations sets, because younger agents focus on a subset of the observations used by older agents

Model

- Economy populated by OLGs of risk-neutral, ex-ante identical, traders
- Traders remain on the market with probability ϕ, leave with $1 - \phi$
- Trade a single divisible stock (S&P500), which is in fixed supply
- There are bounds on the minimum and the maximum investment in the asset

Model: preferences and constraints

$$\max_{S_{it}} E_0^i \sum_{t=0}^{\infty} \beta^t C_{it} \tag{1}$$

subject to

$$C_{it} + P_t S_{it} \leq (P_t + D_t) S_{it-1} + Y_t \tag{2}$$

$$\underline{L}_t \leq P_t S_{it} \leq \bar{L}_t \tag{3}$$

$$\underline{L}_t = 0, \quad \bar{L}_t = \lambda D_t > 0$$

where D_t is given

First-order optimality conditions

$$P_{it} = \beta E_{it} \left(P_{t+1} + D_{t+1} \right) \tag{4}$$

is individual i's "reservation price", so that

$$\begin{aligned}
&\text{if} \quad P_t < P_{it}, \quad \text{then } P_t S_{it} = \bar{L}_t && \text{(5a)} \\
&\text{if} \quad P_t > P_{it}, \quad \text{then } P_t S_{it} = \underline{L}_t && \text{(5b)} \\
&\text{if} \quad P_t = P_{it}, \quad \text{then } P_t S_{it} \in [\underline{L}_t, \bar{L}_t] && \text{(5c)}
\end{aligned}$$

Symmetric rational expectations equilibrium

Removing subscript i

$$P_t = \beta E_t \left(P_{t+1} + D_{t+1}\right) \tag{6}$$

Iterating forward

$$P_t^{RE} = \frac{\beta \exp(\mu + \sigma^2/2)}{1 - \beta \exp(\mu + \sigma^2/2)} D_t \tag{7}$$

Price-dividend ratio

$$\frac{P_t^{RE}}{D_t} = const \tag{8}$$

Everybody agrees on the RE price, there is no trade

Learning from experience

- Individuals learn the stochastic properties of dividends and of stock prices
- When an individual retires, he is replaced by a newborn who inherits the individual's assets but not his accumulated "wisdom"
- Newborns start learning from scratch
- Individuals only consider data realized during their own lives,

$$I_t^s = \{P_\tau, D_\tau\}_{\tau=t-s}^{t} \tag{9}$$

Learning from experience

Agents receive information on stock price and dividend growth

$$x_t = \begin{bmatrix} \log(P_t/P_{t-1}) \\ \log(D_t/D_{t-1}) \end{bmatrix} = \begin{bmatrix} m^P \\ m^D \end{bmatrix} + \begin{bmatrix} \epsilon_t^P \\ \epsilon_t^D \end{bmatrix}$$

And estimate m^P and m^D recursively following

$$m_{s,t} = m_{s-1,t-1} + \gamma_{s,t}\left(x_t - m_{s-1,t-1}\right) \tag{10}$$

with gain (as in Malmendier-Nagel)

$$\gamma_s = \begin{cases} \frac{\theta}{s}, & \text{if } s \geq \theta \\ 1, & \text{if } s < \theta \end{cases} \tag{11}$$

where s is age and θ determines the shape of the function of weights on past information

Shape of gain and weights on past information

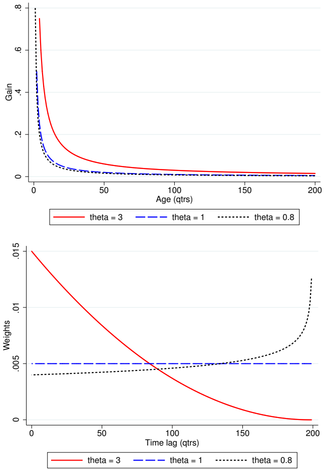


Figure 2: Examples of gain sequences (top) and associated implied weighting of past data (bottom) for an individual who is 50 years (200 quarters) old. The top panel shows the sequence of gains as a function of age. The bottom panel shows the weighting of past data implied by the gain sequence in the top panel, with the weights for most recent data to the left and weights for early-life experiences to the right.

Equililbrium

Definition

An internally rational expectations equilibrium (Adam and Marcet, 2011) consists of a sequence of equilibrium price functions $\{P_t\}_{t=0}^{\infty}$, where $P_t : \Omega_D^t \to R_+$ for each t, contingent choices $\{C_{it}, S_{it}\}_{t=0}^{\infty}$ where $(C_{it}, S_{it}) : \Omega^t \to R^2$, and probability beliefs Π_i for each agent i, such that

1. All agents choose a function (C_{it}, S_{it}) to maximize their expected utility subject to the budget constraint, taking as given the probability measure Π_i.
2. Markets clear

Calibration

- Monthly series for dividends are taken from Shiller for 1871-2014
- $\beta = 0.9977$ consistent with an average real interest rate of 2.7%
- $\phi = 0.9979$ consistent with average life on the market of 40 years
- $\lambda = 480$ to match an average stock market participation rate of 65%
- $\theta = 3.044$ from Malmendier and Nagel (2013)

Fixed point algorithm for P_t

For each point in time $t = 1, ..., T$:

1. Start with P_{t-1} as initial guess for the price
2. Update the price beliefs for each cohort

$$m^P_{s,t} = m^P_{s-1,t-1} + \gamma_s \left[\log(P^{guess}_t / P_{t-1}) - m^P_{s-1,t-1} \right]$$

3. Compute the reservation prices of all cohorts

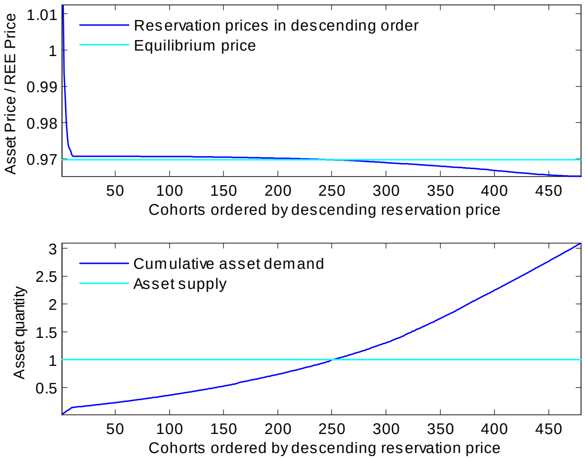
$$P_{s,t} = \beta\phi \left[e^{m^P_{s,t}} P^{guess}_t + e^{m^D_{s,t}} D_t \right]$$

4. Sort reservation prices in decreasing order and index them by j
5. Proceeding from the highest reservation price, find the reservation price of the marginal cohort n, $P^*_t = P_{n,t}$, such that

$$\sum_{j=0}^{n-1} f_j \bar{S}_t < 1, \text{ and } \sum_{j=0}^{n} f_j \bar{S}_t \geq 1$$

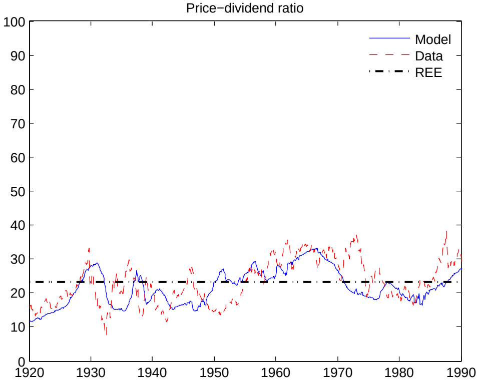
6. Update the guess and repeat until convergence, $P^{guess}_t = P^*_t$

Computation of equilibrium price



Distribution of reservation prices and asset demand across cohorts

Price-dividend ratio



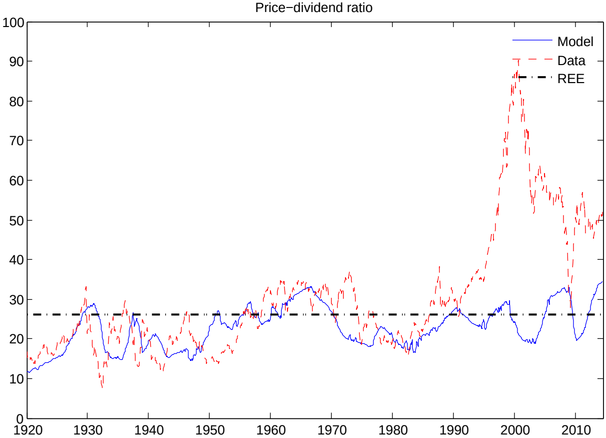
Oscillating dynamics: momentum

- Two elements responsible for the oscillating dynamics:
- Momentum, rooted in the infrequent resetting of the learning of newborns
- A fraction of young individuals enters the market whose learning path is more strongly influenced by the more recent stock price and dividend realizations.
- Their forecasts inform their trading activities, and, through trade, affect the realized stock price, pulling the beliefs of older generations toward the more recent price change realizations.

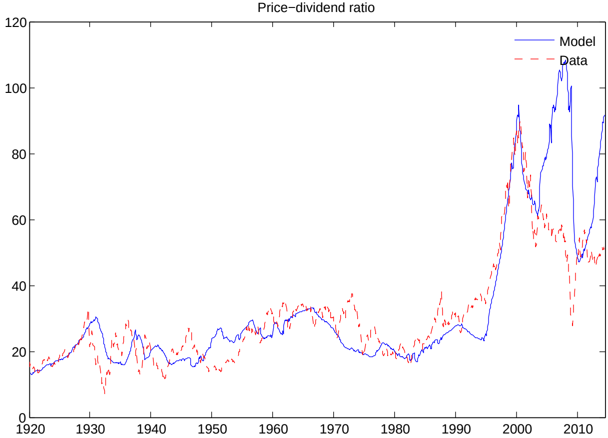
Oscillating dynamics: trend reversion

- Trend-reversion, emanating from the constraints on individual risky asset exposure
- As the stock price rises far above the REE, optimistic investors can afford to buy less shares.
- Because, in equilibrium, all shares must be held by someone, the stock price has to fall to the valuation of less optimistic investors.
- The same reflecting force operates “from below”, when the stock price falls too far beneath the REE
- The combination of the two factors – momentum and trend reversion – results in boom-and-bust cycles that are only loosely related to dividends.

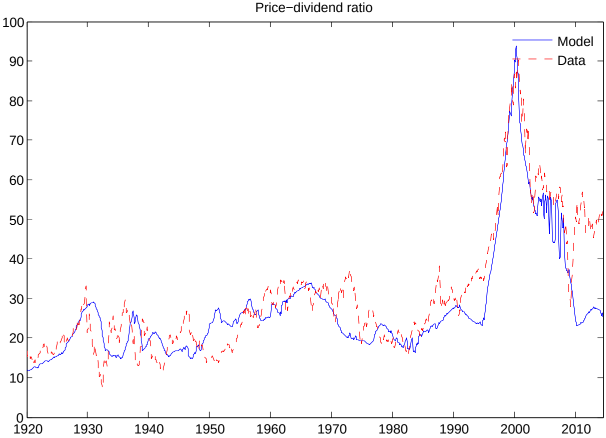
Price-dividend ratio dynamics



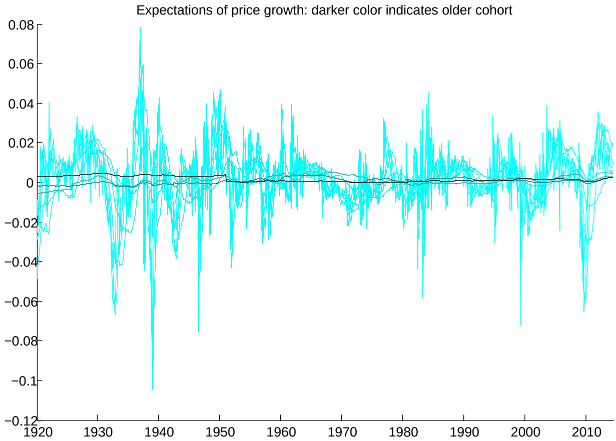
Price-dividend cycles: a rise in investment constraint λ



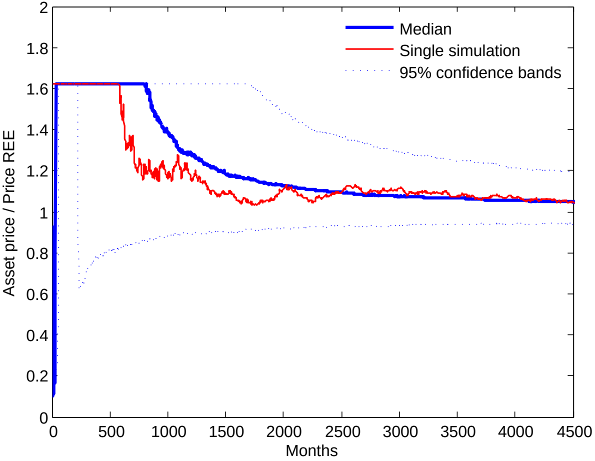
Price-dividend cycles: a rise and drop in λ



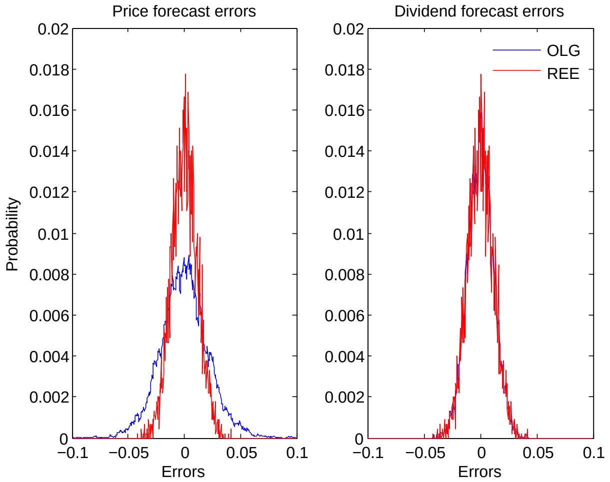
Disagreement: expectations of young are more volatile



Convergence to REE: optimal learning from all data



Errors: OLG model with learning vs REE



Conclusions

- If people learn optimally from all historical data then they can reach a REE (even if they know nothing about each other)
- However, if they don't learn from all the data, then bubbles emerge
- Can policymakers identify and moderate such bubbles?