Optimal monetary policy with state-dependent pricing

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Introduction

- What is a good monetary policy when competition is monopolistic and prices are sticky?
- A large literature studies this under Calvo pricing
- Timing of price changes in principle not independent of policy (e.g. Golosov-Lucas, 2007)
- Our goal: extend analysis to state-dependent pricing

This paper

- Stochastic menu costs as in Dotsey, King and Wolman (1999)
  (fixed costs is a special case)
- Inefficient steady-state: no subsidy offsets the markup distortion from monopolistic competition
  (unlike CGG, 1999, Woodford, 2002, or Yun, 2005)
- The rest is standard (e.g. as in Benigno and Woodford, 2005)
- The central bank sets the nominal interest rate;
  money is just a unit of account

Some caveats

- Optimal policy from a "timeless perspective":
  ignores incentives to behave differently in the initial periods
- Abstract from idiosyncratic shocks to desired prices;
  consider idiosyncratic shocks only to adjustment costs
- Abstract from other distortions: no sticky wages or such
- A theoretical exploration of models;
  no claim about real-world optimal policy

Preview of the results

- If preferences are isoelastic and there is no government spending, it is optimal to commit to zero inflation both in the long run and in reaction to shocks.
- Holds for a general specification of the menu cost distribution
- Optimal allocation: price markups are positive but constant, output is at its natural level, and price dispersion is minimized
- This prescription coincides with the one obtained under Calvo.

Intuition (1/2)

- Relative to Calvo, two additional welfare effects of inflation:

    1. "Menu costs" wasted on changing prices:
    minimized at zero inflation

    2. Price adjustment frequency is endogenous:
    the central bank could have an incentive to use inflation to affect the rate at which firms reoptimize

- "Envelope property": adjusting firms choose prices optimally, so a marginal deviation of inflation from zero has no effect on profits and hence has no effect on the rate of adjustment.

Intuition (2/2)

- The same reasons for which zero inflation is optimal under Calvo pricing continue to hold under stochastic menu costs
- Inefficient price dispersion is minimized at zero inflation
- The marginal welfare gain from raising output towards its efficient level (a movement along the NKPC) exactly cancels out with the marginal welfare loss from generating expectations of future inflation (a shift of the NKPC).

Related literature on optimal monetary policy

1. In the Calvo model: Clarida et. al. (1999), Woodford (2002), Yun (2005), Benigno and Woodford (2005)

2. In a state-dependent model: Lie (2009)

   a. Considers a monetary distortion, which implies a negative long-run rate of inflation

   b. Finds that in the stochastic menu cost model it is desirable to let inflation vary more than with Calvo pricing

Model

Households

A representative household maximizes the expected flow of period utility $u\left(C_t\right) - x\left(N_t; \chi_t\right)$, discounted by β, subject to

$$\int_0^1 P_{it} C_{it} di + R_t^{-1} B_t = W_t N_t + B_{t-1} + \Pi_t$$

$$\begin{aligned} C_t &= \left(\int_0^1 C_{it}^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)} \\ P_t &\equiv \left(\int_0^1 P_{it}^{1-\epsilon} di\right)^{1/(1-\epsilon)} \end{aligned}$$

Model

Firms

- The firm's production function is $y_{it} = z_t n_{it}$
- Labor demand is $n_{it} = y_{it}/z_t$ and real marginal cost is w_t/z_t
- Demand for individual goods $y_{it} = \left(P_{it}/P_t\right)^{-\epsilon} y_t$

Model

Menu cost shocks

- Firms face random lump-sum costs of adjusting prices distributed *i.i.d.* across firms and over time
- Denote the *c.d.f.* and the *p.d.f.* of the stochastic menu cost by $G(\kappa)$ and $g(\kappa)$
- Assuming κ is measured in units of labor time, the total cost paid by a firm changing its price is $w_t\kappa$

Model

Adjustment decision

- Let v_{0t} denote the value of a firm that adjusts its price in period t *before subtracting the menu cost*
- Let $v_{jt}(P)$ denote the value of a firm which has kept its nominal price unchanged at the level P in the last j periods
- The firm will change its price only if $v_{0t} - w_t \kappa > v_{jt}(P)$
- Therefore, from each $j = 1, ..., J-1$ only firms with a menu cost draw $\kappa \leq (v_{0t} - v_{jt}(P)) / w_t$ will change their price

Model

Firms

The real value of an adjusting firm is given by

$$v_{0t} = \max_{P} \left\{ \Pi_t (P) + \beta E_t \frac{u' (C_{t+1})}{u' (C_t)} \left[\begin{array}{c} G (P) \, v_{0,t+1} - \Xi_{1,t+1} (P) \\ + [1 - G (P)] \, v_{1,t+1} (P) \end{array} \right] \right\},$$

where

$$\begin{aligned} \Pi_t (P) &\equiv \left(\frac{P}{P_t} - \frac{w_t}{z_t} \right) \left(\frac{P}{P_t} \right)^{-\epsilon} Y_t \\ G(P) &= G \left(\frac{v_{0,t+1} - v_{1,t+1} (P)}{w_{t+1}} \right) \\ \Xi_{j+1,t+1} (P) &\equiv w_{t+1} \int_0^{(v_{0,t+1} - v_{j+1,t+1} (P))/w_{t+1}} \kappa g (\kappa) \, dk \end{aligned}$$

Model
Firms

The real value of a firm in vintage j not adjusting in time t

$$v_{jt}(P) = \Pi_t(P) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[\begin{array}{c} G(P)\, v_{0,t+1} - \Xi_{j+1,t+1}(P) \\ +[1 - G(P)]\, v_{j+1,t+1}(P) \end{array} \right]$$

where the max operator now is absent.

Model

Two assumptions

We make two technical assumptions:

1. J periods after the last price adjustment, firms draw a zero menu cost and adjust their price
2. the cdf of menu costs has a positive mass at zero

- Assumption 1 makes the state-space finite
- Assumption 2 ensures a unique stationary distribution of firms in the case of zero inflation

Optimal price setting (1/2)

The optimal price-setting decision is given by

$$0 = \Pi_t'\left(P_t^*\right) + \beta E_t \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} \left[1 - G\left(\frac{v_{0,t+1} - v_{1,t+1}(P_t^*)}{w_{t+1}}\right)\right] v_{1,t+1}'\left(P_t^*\right)$$

where

$$\Pi_t'\left(P\right) = \left[\epsilon \frac{w_t}{z_t} - \left(\epsilon - 1\right) \frac{P}{P_t}\right] \left(P\right)^{-\epsilon-1} P_t^{\epsilon} Y_t$$

Optimal price setting (2/2)

The optimal price is

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t u'\left(C_{t+j}\right) \prod_{k=1}^{j}\left(1 - \lambda_{k,t+k}\right) P_{t+j}^{\epsilon} Y_{t+j}\left(w_{t+j}/z_{t+j}\right)}{\sum_{j=0}^{J-1} \beta^j E_t u'\left(C_{t+j}\right) \prod_{k=1}^{j}\left(1 - \lambda_{k,t+k}\right) P_{t+j}^{\epsilon-1} Y_{t+j}}$$

where

$$\lambda_{jt} \equiv G\left(\left(v_{0t} - v_{jt}(P_{t-j}^*)\right)/w_t\right)$$

Like in Calvo, with $\prod_{k=1}^{j}\left(1 - \lambda_{k,t+k}\right)$ replacing $\left(1 - \lambda^{Calvo}\right)^j$

Market clearing (1/2)

- Labor is used both for production and for changing prices
- Aggregate labor demand for production purposes is $\Delta_t Y_t / z_t$, where $\Delta_t \equiv \int_0^1 (P_{it}/P_t)^{-\epsilon} di$ is relative price dispersion.
- Aggregate labor demand for pricing purposes is $\sum_{j=1}^{J-1} \psi_{jt} \int_0^{(v_{0t}-v_{jt})/w_t} \kappa g(\kappa) \, dk$.

Market clearing (2/2)

- Equilibrium in the labor market therefore implies,

$$N_t = \frac{Y_t \Delta_t}{z_t} + \sum_{j=1}^{J-1} \psi_{jt} \int_0^{(v_{0t} - v_{jt})/w_t} \kappa g(\kappa)\, dk.$$

- Equilibrium in the goods market

$$Y_t = C_t + G_t,$$

where G_t follows an exogenous process

Price level dynamics

- Absent firm-level shocks to desired prices, all firms adjusting at time t choose the same nominal price P_t^*
- Let ψ_{jt} denote the time-t fraction of firms with beginning-of-period nominal price P_{t-j}^*
- The price level evolves according to

$$P_t^{1-\epsilon} = \left(P_t^*\right)^{1-\epsilon} \sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(P_{t-j}^*\right)^{1-\epsilon} \left(1 - \lambda_{jt}\right) \psi_{jt},$$

where λ_{jt} are endogenous price adjustment probabilities

Price dispersion and price distribution dynamics

- Price dispersion follows

$$\Delta_t = (p_t^*)^{-\epsilon} \sum_{j=1}^{J} \lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1} \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1} \pi_{t-k}} \right)^{-\epsilon} (1-\lambda_{jt})\,\psi_{jt},$$

- The distribution of beginning-of-period prices evolves according to

$$\psi_{j,t} = (1-\lambda_{j-1,t-1})\,\psi_{j-1,t-1}$$

for $j = 2, ..., J$, and

$$\psi_{1t} = 1 - \sum_{j=2}^{J} \psi_{j,t}$$

Equilibrium

- There are $3J+7$ stationary endogenous variables:
  C_t, N_t, Y_t, R_t, π_t, p_t^*, w_t, Δ_t, $\{\psi_{jt}\}_{j=1}^{J}$, $\{v_{jt}\}_{j=0}^{J-1}$, $\{\lambda_{jt}\}_{j=1}^{J-1}$
- And there are $3J+6$ equilibrium conditions
- What is missing is the specification of monetary policy
- If we used a Taylor rule, this would give us $3J+7$ equations
- Instead, we study optimal monetary policy
  (which doubles the number of equations and variables)

Optimal monetary policy problem: Lagrangean

Special case: $J = 2$ cojorts, $U_t = log(C_t) - \chi_t N_t$, $G_t = 0$

$$
\begin{aligned}
L_0 = {} & E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log(Y_t) - \chi_t \frac{Y_t \Delta_t}{z_t} - \chi_t \psi_t \int_0^{(\tilde{v}_{0t} - \tilde{v}_t)/\chi} \kappa g(\kappa)\, d\kappa \\
& + \phi_t^{p^*} \left[p_t^* \left(1 + \beta (1 - \lambda_{t+1}) \pi_{t+1}^{\epsilon - 1}\right) - \frac{\epsilon}{\epsilon - 1} \left(\frac{\chi_t Y_t}{z_t} + \beta (1 - \lambda_{t+1}) \pi_{t+1}^{\epsilon} \frac{\chi_t Y_{t+1}}{z_{t+1}} \right) \right. \\
& + \phi_t^{\pi} \left[(p_t^*)^{1-\epsilon} (\lambda_t \psi_t + 1 - \psi_t) + \left(\frac{p_{t-1}^*}{\pi_t} \right)^{1-\epsilon} (1 - \lambda_t) \psi_t - 1 \right] \\
& + \phi_t^{\Delta} \left[(p_t^*)^{-\epsilon} (\lambda_t \psi_t + 1 - \psi_t) + \left(\frac{p_{t-1}^*}{\pi_t} \right)^{-\epsilon} (1 - \lambda_t) \psi_t - \Delta_t \right] \\
& + \phi_t^{\lambda} \left[\lambda_t - G\left(\frac{\tilde{v}_{0t} - \tilde{v}_t}{\chi_t} \right) \right] + \phi_t^{\psi} \left[\psi_t + (1 - \lambda_{t-1}) \psi_{t-1} \right] \\
& + \phi_t^{v_0} \left[\left(p_t^* - \frac{\chi_t Y_t}{z_t} \right) (p_t^*)^{-\epsilon} - \tilde{v}_{0t} + \beta \left(\lambda_{t+1} \tilde{v}_{0,t+1} + (1 - \lambda_{t+1}) \tilde{v}_{t+1} - \chi_t \int_0^{(\tilde{v}_{0,t}} \right. \right. \\
& + \phi_t^{v} \left[\left(\frac{p_{t-1}^*}{\pi_t} - \frac{\chi_t Y_t}{z_t} \right) \left(\frac{p_{t-1}^*}{\pi_t} \right)^{-\epsilon} - \tilde{v}_t + \beta \tilde{v}_{0,t+1} \right] \Bigg\} .
\end{aligned}
$$

Solution (1/3)

- Derive FOCs
- Conjecture that $\pi_t = 1$
- It follows that $p_t^* = \Delta_t = 1$
- Both vintages have the same relative price
- Both vintages have the same value, $v_{0t} = v_t$
- Adjustment rate $\lambda_t = G(0) \equiv \bar{\lambda} > 0$
- The vintage distribution converges to $\psi_t = 1/\left(2 - \bar{\lambda}\right) \equiv \bar{\psi}$
- Real marginal cost $=$ 1/markup $= \chi_t Y_t / z_t = (\epsilon - 1)/\epsilon$
- Output equals its flexible-price level

Solution (2/3)

- Impose the conjecture into FOCs
- Solve for the Lagrange multipliers
- Terms involving the Lagrange multipliers $\phi_t^{v_0}$ and ϕ_t^{v} drop out from FOCs w.r.t inflation and the optimal price
- I.e., a marginal deviation of inflation from zero has no effect on adjustment gains and hence no effect on the frequency of price adjustment (the "envelope property")

Solution (3/3)

- In the FOCs w.r.t. inflation and the optimal price there is a positive term and a negative term associated with inflation
- At zero inflation, the gain from a movement along the NKPC is exactly offset by the loss from a shift of the NKPC
- The optimal plan involves no attempt to correct the static markup distortion
- This is true also with Calvo price setting
(but carries over to SDP)

Intuition

There are four potential inefficiencies in the model:

1. the level and volatility of price dispersion
2. the waste of resources due to menu costs
3. the volatility of the average markup
4. the level of the average markup

- The optimal policy does nothing about (4)
- Distortions (1) to (3) are directly related to the friction in price-setting. Absent idiosyncratic shocks to desired prices, "price stability" eliminates all three

Optimal policy with positive government spending

- $u(C_t) = C_t^{1-\gamma}/(1-\gamma)$ and $x(N_t) = \chi N_t^{1+\varphi}/(1+\varphi)$
- with CRRA $\gamma = 2$, $\chi = 6$, $\varphi = 1$
- discount $\beta = 1.04^{-1/4}$ and elasticity $\epsilon = 7$
- $\bar{G} = 0.1$, government spending around 17% of GDP
- Cumulative distribution of menu costs $G(\kappa) = \frac{\xi+\kappa}{\alpha+\kappa}$ with $\xi \to +0$ and $\alpha > 0$
- Fraction of vintage-j firms that adjust their price in time t

$$\lambda_{jt} = G\left(\frac{v_{0t} - v_{jt}}{w_t}\right) = \frac{\xi + (v_{0t} - v_{jt})/w_t}{\alpha + (v_{0t} - v_{jt})/w_t} \geq \frac{\xi}{\alpha}$$

- $\xi = 1e - 10$ and $\alpha = 0.0006$ so that with 2% inflation the average frequency of price changes is 33% per quarter
- Maximum price duration $J = 24$ quarters

Hazard rate and vintage distribution at 2% inflation

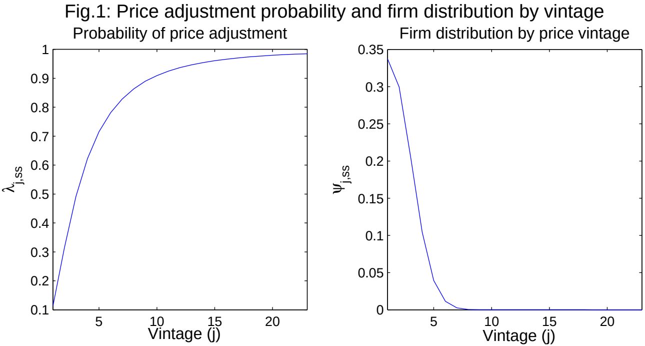
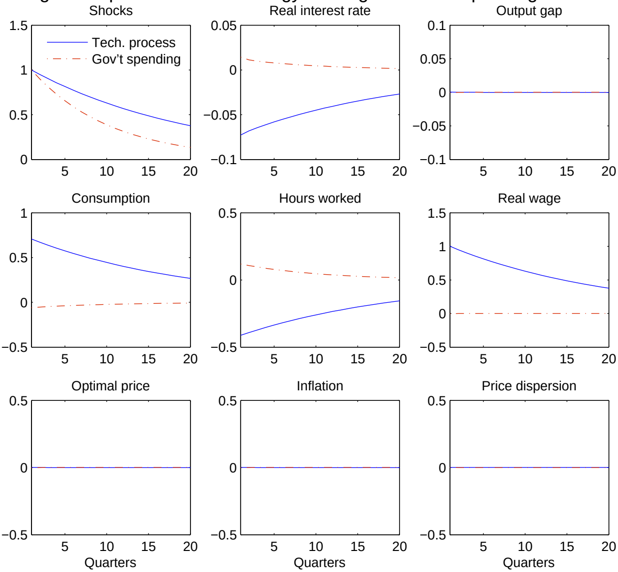


Fig.2: Responses to a technology and a government spending shock



Conclusions

- The main lessons for optimal policy derived in the Calvo model carry over to a more general setup in which the probability of changing prices depends on the state of the economy
- The optimal long run rate of inflation is zero, and the short run objective is "price stability"
- Support for using the Calvo model for optimal monetary policy analysis despite its apparent conflict with the Lucas critique

Left for future research

- A fuller model would have firm-level shocks to desired prices, e.g. idiosyncratic productivity shocks
- In general monetary policy would not be able to replicate the flexible price allocation "for free"
- Monetary policy could be very volatile and induce continous price adjustment by all firms but this is likely suboptimal
- Local deviations from price stability are unlikely to affect significantly the rate of adjustment
- Will shift the balance between fraction of price decreases and fraction of price increases: welfare gains?