Distributional dynamics under smoothly state-dependent pricing

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Motivation

- Sticky prices are a key ingredient in modern macro models
  - How best to model price stickiness?
  - Individual price stickiness $\Rightarrow$ rigidity of the aggregate price level?
- Calvo (1983): adjustment probability is an exogenous constant
  - Analytical tractability
  - Monetary shocks have large and persistent real effects
- Golosov-Lucas (2007): fixed "menu" cost + idiosyncratic shocks
  - Calibrated to match the standard deviation of price changes
  - Strong selection effect $\Rightarrow$ near-neutrality of money

Gist of this paper

- It calibrates and simulates a general framework of "smoothly state-dependent pricing"
- It nests the Calvo and the fixed menu cost (FMC) models as two opposite limiting cases
- Premise: price adjustment is more likely when it is more valuable

$$\lambda = \lambda(\Delta V), \quad \lambda' \geq 0, \quad 0 \leq \lambda \leq 1$$

Main result

- We postulate a parametric family for λ which nests Calvo and FMC
- Discipline is imposed by fitting the model to the size distribution of price changes from recent US retail microdata
- One of the estimated parameters controls "state dependence"
- Matching the smooth distribution of price changes found in the data requires rather low state dependence
- Result: monetary shocks have substantial real effects, only slightly weaker than the Calvo model

Other contributions

- GL07 studied *iid* money shocks; we study also autocorrelated shocks
  - The shape and persistence of responses is determined by the degree of state dependence, not by the autocorrelation of the shocks
- Impulse-responses under a Taylor rule
  - Reinforces the finding of non-trivial real effects of nominal shocks

Other contributions

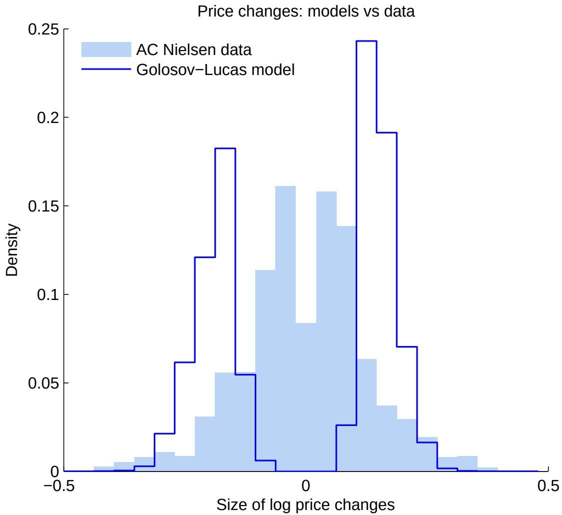
- Inflation decomposition into "intensive margin", "extensive margin", and "selection effect"
  - Vindicates the claim of GL07 that the selection effect is crucial for the behavior of the FMC model
- Comparison of true and estimated price responses to firm-level shocks
  - The true response fades in and out gradually
  - But one popular estimation procedure wrongly suggests that firm-level shocks have an immediate and permanent impact on prices

Outline of the talk

Related literature: state-dependent pricing

- Previous work obtained solutions by limiting the scope of analysis
  - Partial equilibrium (Caballero-Engel, 2007; Klenow-Kryvtsov, 2008)
  - Only aggregate shocks (Dotsey-King-Wolman, 1999)
  - Strong assumptions about the idiosyncratic process (Caplin-Spulber, 1987; Gertler-Leahy, 2005)
- But firms frequently hit by large idiosyncratic shocks (K&K, 2008)
  - Such shocks could greatly affect firms' incentives to adjust prices, and may not "wash out" in the aggregate
- Golosov-Lucas (2007): a general equilibrium menu cost model with firm-level shocks
  - Striking near-neutrality result, but model's fit to price data questionable

Histogram of non-zero price changes



Related literature: distribution of price changes

- Three proposals to "fix" the distribution
  - Sectoral heterogeneity in fixed menu costs (Klenow-Kryvtsov, 2008)
  - Multiple products on the same "menu", combined with leptokurtic technology shocks (Midrigan, 2010)
  - A mix of flexible- and sticky-price firms, plus a mix of two distributions of productivity shocks (Dotsey-King-Wolman, 2008)

Related literature: distribution of price changes

- Our approach is simpler: assume the probability of adjustment increases with the gain, treating the hazard function as a primitive
  - One interpretation is "stochastic menu costs" like Dotsey et al.
  - Alternatively, "near-rational behavior" like Akerlof and Yellen (1985)
- We match the distribution at least as well with less free parameters
- Key is that the hazard increases smoothly with the gain
- Smoothness is the same property that mitigates the strong selection effect found by GL07

Related literature: other micro facts

- Sales
  - Eichenbaum-Jaimovich-Rebelo (2008) and Kehoe-Midrigan (2010): "temporary" price changes cheaper than other price changes
  - Guimaraes-Sheedy (2010): sales as "stochastic price discrimination"
- Conclude that sales have little relevance for monetary transmission, which depends on the frequency of "regular" price changes.
- Our model has no motive for sales; hence, we compare it to a dataset of regular price changes

Related literature: other micro facts

- Differential responses to aggregate vs. disaggregate shocks
  - Boivin-Giannoni-Mihov (2009) and Mackowiak-Moench-Wiederholt (2009) compare the responses to sectoral and aggregate shocks
  - Estimate that prices respond quickly to idiosyncratic shocks but sluggishly to aggregate shocks
- We show with a Monte Carlo exercise that this finding should be treated with caution
  - It may be the result of confusing price changes due to idiosyncratic factors with delayed responses to aggregate shocks

Model: households

Period utility

$$\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \chi N_t + \nu \log(M_t/P_t)$$

Consumption basket

$$C_t = \left\{ \int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}$$

Period budget constraint

$$\int_0^1 P_{it} C_{it} di + M_t + R_t^{-1} B_t = W_t N_t + M_{t-1} + B_{t-1} + T_t + U_t$$

Price index

$$P_t = \left\{ \int_0^1 P_{it}{}^{1-\epsilon} di \right\}^{\frac{1}{1-\epsilon}}$$

Model: monopolistic firms

Output technology

$$Y_{it} = A_{it} N_{it}$$

Idiosyncratic productivity process

$$\log A_{it} = \rho \log A_{it-1} + \varepsilon_{it}^{a}$$

Period profit

$$U(P_{it}, A_{it}, \Omega_t) = P_{it} Y_{it} - W_t N_{it} = \left(P_{it} - \frac{W_t}{A_{it}} \right) C_t P_t^{\epsilon} P_{it}^{-\epsilon}$$

Value function

$$V(P, A, \Omega) = U(P, A, \Omega) + E\{Q(\Omega)\left[V(P, A', \Omega') + G(P, A', \Omega')\right]|A, \Omega\}$$

Model: monopolistic firms

Gain if adjusting

$$D(P, A, \Omega) = \max_{P^*} V(P^*, A, \Omega) - V(P, A, \Omega)$$

Probability of adjustment

$$\lambda = \lambda\left[\frac{D(P, A, \Omega)}{W(\Omega)}\right]$$

Expected gain from adjustment

$$G(P, A', \Omega') \equiv \lambda\left[\frac{D(P, A', \Omega')}{W(\Omega')}\right] D(P, A', \Omega')$$

Adjustment function

Probability of adjustment

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1 - \bar{\lambda})\left(\frac{\alpha}{L}\right)^{\xi}}$$

where $L \equiv D/W$, $\alpha > 0$, $\xi > 0$, and $\bar{\lambda} \in (0, 1)$

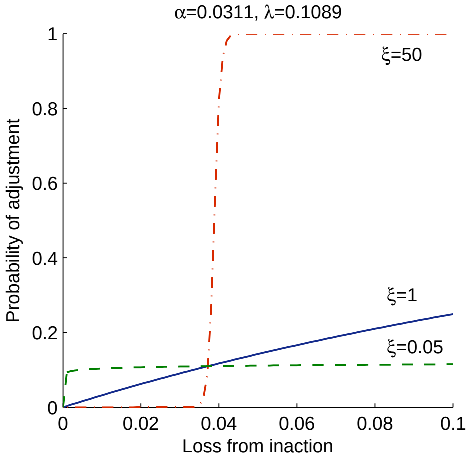
- Satisfies $\lambda' \geq 0$ and $\leq \lambda \leq 1$
- Parameter ξ controls the degree of state dependence
- $\lambda(L)$ is concave for $\xi \leq 1$ and S-shaped for $\xi > 1$

Nesting Calvo and fixed menu costs

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda}+(1-\bar{\lambda})\left(\frac{\alpha}{L}\right)^{\xi}}$$

$$\stackrel{\xi \to 0}{\longrightarrow} \bar{\lambda}$$

$$\stackrel{\xi \to \infty}{\longrightarrow} \mathbf{1}\{L > \alpha\}$$



Nesting alternative sticky price models

Table 1: Adjustment specifications

Specification	Adjustment probability $\lambda(L)$	Mean gains, in units of time: $G(P,A,\Omega)/W(\Omega)$
Calvo	$\bar{\lambda}$	$\bar{\lambda}L(P,A,\Omega)$
Fixed MC	$\mathbf{1}\{L \geq \alpha\}$	$\lambda(L(P,A,\Omega))\,[L(P,A,\Omega)-\alpha]$
Woodford	$\bar{\lambda}/[\bar{\lambda}+\left(1-\bar{\lambda}\right)\exp(\xi(\alpha-L))]$	$\lambda(L(P,A,\Omega))\,L(P,A,\Omega)$
Stoch. MC	$\bar{\lambda}/[\bar{\lambda}+\left(1-\bar{\lambda}\right)(\alpha/L)^{\xi}]$	$\lambda(L(P,A,\Omega))\,[L(P,A,\Omega)-E\left(\kappa \mid \kappa<\lambda\left(L(P,A,\Omega)\right)\right)]$
SSDP	$\bar{\lambda}/[\bar{\lambda}+\left(1-\bar{\lambda}\right)(\alpha/L)^{\xi}]$	$\lambda(L(P,A,\Omega))\,L(P,A,\Omega)$

Note: $\lambda(L)$ is the probability of price adjustment; L is the real loss from failure to adjust, as a function of firm's price P and productivity A, and aggregate conditions Ω. G represents mean nominal gains from adjustment; dividing by the nominal wage W converts gains to real terms. $\bar{\lambda}$, α and ξ are parameters to be estimated.

Model: monetary policy and aggregate consistency

Two specifications for monetary policy:

1. $M_t/M_{t-1} = \mu^* \exp(z_t)$
2. $\frac{R_t}{R^*} = \exp(-z_t) \left(\left(\frac{P_t/P_{t-1}}{\Pi^*} \right)^{\phi_\pi} \left(\frac{C_t}{C^*} \right)^{\phi_c} \right)^{1-\phi_R} \left(\frac{R_{t-1}}{R^*} \right)^{\phi_R}$

The systematic component is perturbed by: $z_t = \phi_z z_{t-1} + \epsilon_t^z$

The government budget is balanced each period: $M_t = M_{t-1} + T_t$

Bond market clearing: $B_t = 0$

Model: monetary policy and aggregate consistency

Labor market clearing

$$N_t = \int_0^1 \frac{C_{it}}{A_{it}} di = P_t^{\epsilon} C_t \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di = \Delta_t C_t$$

Measure of price dispersion

$$\Delta_t \equiv P_t^{\epsilon} \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di$$

Aggregate state variables

1. $\Omega \equiv (z_t, R_{t-1}, \Phi_{t-1})$, or
2. $\Omega \equiv (z_t, M_{t-1}, \Phi_{t-1})$

where Φ_{t-1} is the distribution of firms on (p, A)

Computation

- In general, need to keep track of the entire distribution of firms, an infinite-dimensional object
- Reiter's (2009) "projection & perturbation" method, combines linearity and nonlinearity
- Appropriate for the context of price setting: idiosyncratic shocks are relatively large; aggregate shocks are relatively small
  1. Compute steady-state by non-linear projection on a finite grid
  2. Compute aggregate dynamics by linearization around each grid point
- Bellman eq.: a large system of expectational difference equations

Computation: steady-state (projection)

- Aggregate steady state: no aggregate, only idiosyncratic shocks
- Real prices converge to an ergodic distribution Ψ

1. Guess real wage: w
2. Consumption: $C = (\chi/w)^{1/\gamma}$
3. Payoff at grid points: $U_{ij} = (p_i - w/A_j)\, C p_i^{-\epsilon}$
4. Iterate on value matrix: $\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' (\mathbf{V} + \mathbf{G}) \mathbf{S}$
5. Iterate on distribution matrices:
   - $\tilde{\mathbf{\Psi}} = \mathbf{R}\mathbf{\Psi}\mathbf{S}'$
   - $\mathbf{\Psi} = (\mathbf{1}_{\#^p\#^a} - \mathbf{\Lambda}) .* \tilde{\mathbf{\Psi}} + \mathbf{P}^*_{\#^p\#^a} .* \left(\mathbf{1}_{\#^p\#^p} * (\mathbf{\Lambda} .* \tilde{\mathbf{\Psi}})\right)$
6. Check if $\sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi_t^{jk} p_j^{1-\epsilon} = 1$, and adjust w until it holds

Computation: dynamics (perturbation)

- Dynamic Bellman equation:

$$\mathbf{V}_t = \mathbf{U}_t + \beta E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \mathbf{R}'_{t+1} \left(\mathbf{V}_{t+1} + \mathbf{G}_{t+1} \right) \mathbf{S} \right]$$

- Distributional dynamics:
  - $\tilde{\mathbf{\Psi}}_t = \mathbf{R}_t \mathbf{\Psi}_{t-1} \mathbf{S}'$
  - $\mathbf{\Psi}_t = \left(\mathbf{1}_{\#^p \#^a} - \mathbf{\Lambda}_t \right) .* \tilde{\mathbf{\Psi}}_t + \mathbf{P}_t^* .* \left(\mathbf{1}_{\#^p \#^a} * \left(\mathbf{\Lambda}_t .* \tilde{\mathbf{\Psi}}_t \right) \right)$
- Collect variables in vector: $X_t = \left(vec(\mathbf{\Psi}_{t-1}), vec(\mathbf{V}_t), C_t, \Pi_t, R_{t-1} \right)$
- Model: $E_t \mathcal{F} \left(X_{t+1}, X_t, z_{t+1}, z_t \right) = 0$
- Linearization: $E_t \mathcal{A} \Delta X_{t+1} + \mathcal{B} \Delta X_t + E_t \mathcal{C} z_{t+1} + \mathcal{D} z_t = 0$
- Solve with Klein's QZ method for linear RE models

Calibration and estimation

Discount factor	$\beta = 1.04^{-1/12}$	Golosov-Lucas (2007)
CRRA	$\gamma = 2$	Ibid.
Labor supply	$\chi = 6$	Ibid.
MIUF coeff.	$\nu = 1$	Ibid.
Elast. subst.	$\epsilon = 7$	Ibid.
Money growth	$\mu = 1$	AC Nielsen data: zero inflation

The productivity $(\rho, \sigma_{\varepsilon}^2)$ and adjustment parameters $(\bar{\lambda}, \alpha, \xi)$ are estimated by minimizing a distance criterion:

$$\min(25 \left\| fr - 0.10 \right\| + \left\| histM - histD \right\|)$$

Estimation results

	Productivity parameters See eq. (8) for definitions	Adjustment parameters See Table 1 for definitions
Calvo	$(\sigma_\varepsilon, \rho) = (0.0850, 0.8540)$	$\bar{\lambda} = 0.10$
Fixed MC	$(\sigma_\varepsilon, \rho) = (0.0771, 0.8280)$	$\alpha = 0.0665$
Woodford	$(\sigma_\varepsilon, \rho) = (0.0924, 0.8575)$	$(\bar{\lambda}, \alpha, \xi) = (0.0945, 0.0611, 1.3335)$
Stoch. MC	$(\sigma_\varepsilon, \rho) = (0.0676, 0.9003)$	$(\bar{\lambda}, \alpha, \xi) = (0.1100, 0.0373, 0.2351)$
SSDP	$(\sigma_\varepsilon, \rho) = (0.0677, 0.9002)$	$(\bar{\lambda}, \alpha, \xi) = (0.1101, 0.0372, 0.2346)$

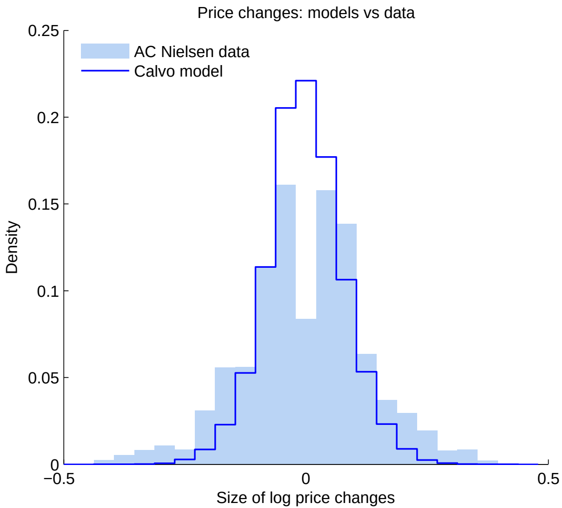
Table 2: Estimated parameters

Simulated moments and evidence

Moments	Model					Evidence			
	Calvo	FMC	Wdfd	SMC	SSDP	MAC	MD	NS	KK
Frequency of price changes	10.0	10.0	10.0	10.0	10.0	20.5	19.2	10	13.9
Mean absolute price change	6.4	17.9	10.3	10.0	10.1	10.5	7.7		11.3
Std of price changes	8.2	18.4	13.6	12.2	12.2	13.2	10.4		
Kurtosis of price changes	3.5	1.3	4.0	2.9	2.9	3.5	5.4		
% price changes ≤5% in abs value	47.9	0.0	37.0	26.3	26.3	25	47		44
Mean loss in % of frictionless profit	36.8	10.6	37.4	25.6	25.6				
Mean loss in % of frictionless revenue	5.2	1.5	5.3	3.6	3.6				
Fit: Kolmogorov-Smirnov statistic	0.111	0.356	0.038	0.024	0.025				
Fit: Euclidean distance	0.159	0.409	0.072	0.060	0.056				

Note: Price statistics refer to non-sale consumer price changes and are stated in percent. The last four columns report statistics from Midrigan (2008) for AC Nielsen (MAC) and Dominick's (MD), Nakamura and Steinsson (2008) (NS), and

Histogram of non-zero price changes



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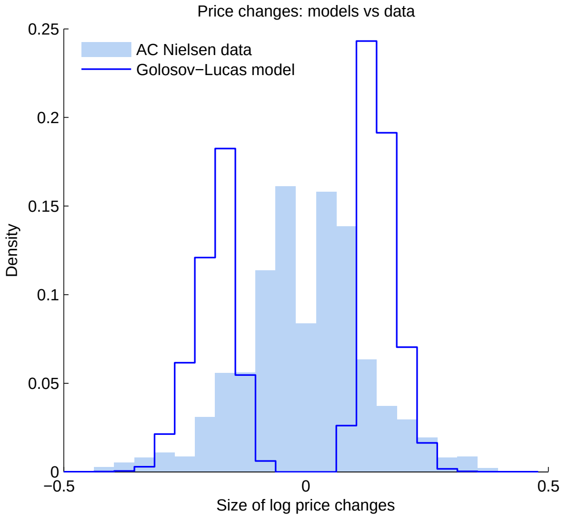

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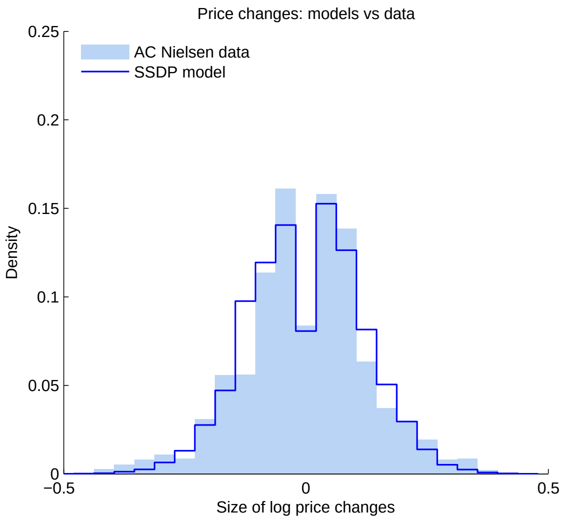
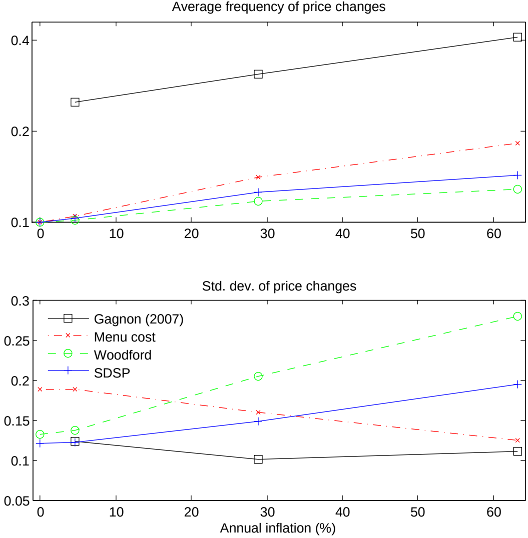


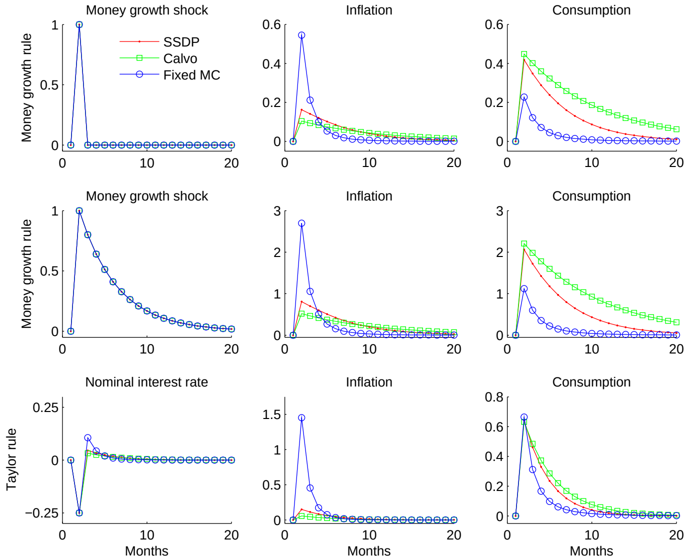
Effects of trend inflation



Responses to monetary policy shocks

- Compare impulse-responses under FMC, Calvo, SSDP
- Shock process: $z_t = \phi_z z_{t-1} + \epsilon_t^z$
- Two policy regimes
  - Money growth rule: $M_t / M_{t-1} = \mu^* \exp(z_t)$
    - Uncorrelated money growth shock, $\phi_z = 0$
    - Correlated money growth shock, $\phi_z = 0.8$
  - Taylor rule: $\frac{R_t}{R^*} = \exp(-z_t) \left(\left(\frac{P_t / P_{t-1}}{\Pi^*} \right)^{\phi_\pi} \left(\frac{C_t}{C^*} \right)^{\phi_c} \right)^{1-\phi_R} \left(\frac{R_{t-1}}{R^*} \right)^{\phi_R}$
    - Coefficients $\phi_\pi = 2$, $\phi_c = 0.5$, $\phi_R = 0.9$
    - Uncorrelated shock, $\phi_z = 0$

Inflation and output responses to monetary policy shocks



Phillips curve regressions

Table 3. Variance decomposition and Phillips curves of alternative models

	Data	SSDP	Calvo	FMC
Std of quarterly inflation (×100)	0.246	0.246	0.246	0.246
% explained by nominal shock		100	100	100
Money growth rule (see eq. 16-17)				
Std of money growth shock (×100)		0.174	0.224	0.111
Std of detrended output (×100)	0.909	0.586	1.053	0.121
% explained by money growth shock		64.5	115.9	13.3
Slope coeff. of the Phillips curve		0.598	1.069	0.134
Standard error		0.004	0.039	0.005
Taylor rule (see eq. 18)				
Std of Taylor rule shock (×100)		0.393	0.918	0.129
Std of detrended output (×100)	0.909	0.995	2.741	0.134
% explained by Taylor rule shock		109.6	301.6	14.7
Slope coeff. of the Phillips curve		1.055	2.785	0.126
Standard error		0.093	0.290	0.006

Inflation decomposition: Klenow-Kryvtsov

Klenow-Kryvtsov's decomposition:

$$\pi_t = \bar{x}_t \bar{\lambda}_t \quad \Rightarrow \quad \Delta\pi_t \approx \bar{\lambda}\Delta\bar{x}_t + \bar{x}\Delta\bar{\lambda}_t$$

where $\bar{x}_t$ is the average price change $\left(\equiv \frac{\sum_{j,k} x_t^{*jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}}{\sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}} \right)$

and $\bar{\lambda}_t$ is the fraction of adjusting firms $\left(\equiv \sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk} \right)$

Average price change is that among *adjusters*: a *self-selected* group

Our inflation decomposition

Inflation identity: $\pi_t = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} x_t^{*jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}$

where $x_t^{*jk} \equiv \log\left(\frac{p_t^*(a^k)}{p^j}\right)$ are firms' *desired* log price changes

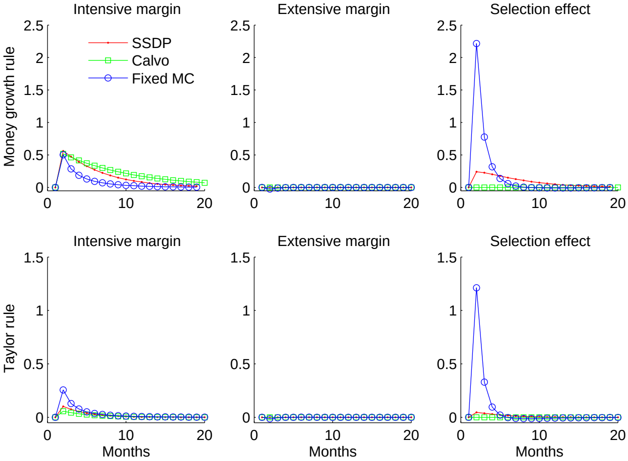
Our decomposition:

$$\pi_t = \bar{x}_t^* \bar{\lambda}_t + \sum_{j,k} x_t^{*jk} \left(\lambda_t^{jk} - \bar{\lambda}_t\right) \widetilde{\Psi}_t^{jk}, \qquad \bar{x}_t^* \equiv \sum_{j,k} x_t^{*jk} \widetilde{\Psi}_t^{jk}$$

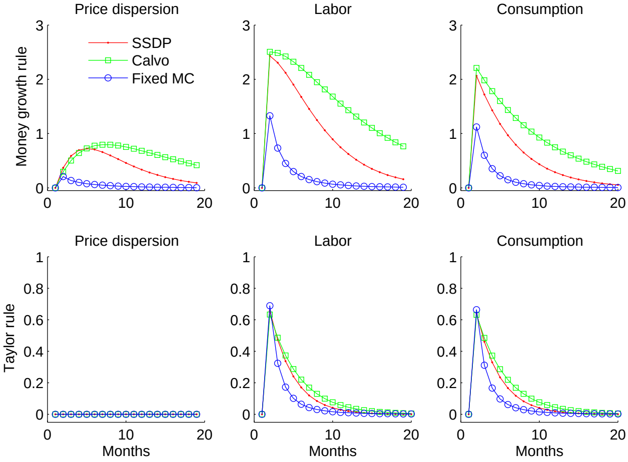
where $\bar{x}_t^*$ is the average *desired* log price change

$$\Delta\pi_t = \bar{\lambda}\Delta\bar{x}_t^* + \bar{x}^*\Delta\bar{\lambda}_t + \Delta\sum_{j,k} x_t^{*jk} \left(\lambda_t^{jk} - \bar{\lambda}_t\right) \widetilde{\Psi}_t^{jk}$$

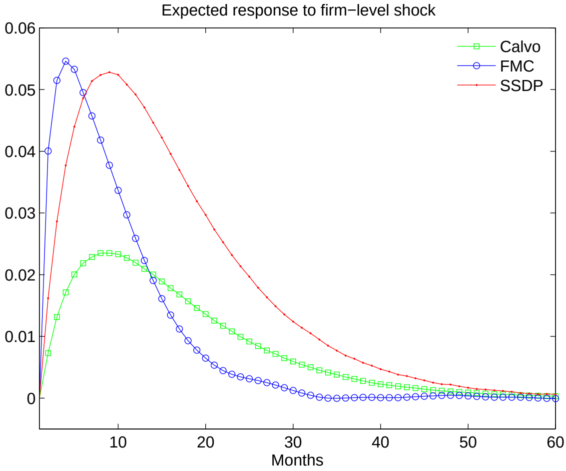
Inflation decomposition



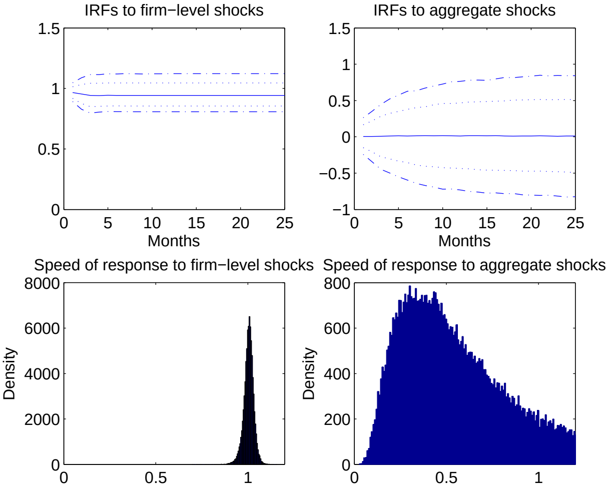
Inefficient price dispersion



Effects of idiosyncratic shocks: true mean responses



Effects of idiosyncratic shocks: estimated responses



Conclusions

- We characterize the firm's distributional dynamics in a general framework of state-dependent pricing
- The estimated model implies that prices rise gradually in response to monetary stimulus, causing a large, persistent rise in consumption
- Across models, the main factor determining how monetary shocks propagate through the economy is the degree of state dependence
- The parameterization most consistent with microdata is fairly close to the Calvo model in terms of the real effects of nominal shocks

Conclusions

- We decompose the response of inflation into an intensive margin, an extensive margin, and a selection effect
- In the baseline model, about two-thirds of the effect of a monetary shock comes through the intensive margin, and around one-third through the selection effect
- The same property which makes money nearly neutral in the FMC model is the one which makes that model inconsistent with micro evidence on price changes
- A model in which adjustment depends more smoothly on the value of adjusting fits microdata better and yields larger real effects of nominal shocks